On the Calculus of Closed Set-Valued Functions

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Communicated by the Editors

1. Introduction. For every t in the real interval $[0, T]$ let $F(t)$ be a subset of the n -dimensional euclidean space E_n . The integral $\int_0^T F(t)dt$ (denoted for simplicity by $\int F$) of the set-valued function F over $[0, T]$ is defined by

$$\int F = \left\{ \int_0^T f(t) dt : f \text{ is an integrable selector of } F \right\}.$$

Here a selector of F means a point-valued measurable function f such that $f(t) \in F(t)$ dt -almost everywhere.

The definition was stated in this form by Aumann [3] and it is a generalization of both the integral of point-valued functions and the summation of sets. This integration concept has been found very applicable in several mathematical fields, especially in control theory and mathematical economics.

In this paper we shall contribute to the basic theory, mainly in the two directions described below.

In [6] Debreu deals with functions that have compact and convex values. The collection of the compact and convex sets can be regarded as a cone in a Banach space, thus a set-valued function is now a point-valued function into this space. Debreu showed that the integration of the set-valued functions can be done as the Bochner integration into the Banach space. Debreu developed the theory for subsets of a normed space. We shall deal only with E_n , but will show that the integration of other classes of functions (rather than the compact-valued) can be done in a similar way. Also our embedding will be much simpler. This will be done in Section 3.

In Section 4 we shall deal with the indefinite integral, *i.e.* with the set-valued function $U(t) = \int_0^t F(\tau)d\tau$. The main theorem here is a necessary and sufficient condition for a compact set-valued function to be an indefinite integral of a certain function. The techniques involved will be used in order to present generalizations and to simplify proofs of results obtained by Hermes [8] and Bridgland [5]. In these papers the readers can find the motivation for dealing with the indefinite integral.

A main tool in our paper is the support function of a set-valued function. We shall give some preliminary results in Section 2.

The following two properties are basic to the theory and will be used in the sequel several times. 1. *The integral of a set-valued function is a convex set.* This follows since we are integrating with respect to a nonatomic measure, see [3]. 2. *If the graph of F i.e. the set $\{(t, x): x \in F(t)\}$ belongs to the smallest σ -algebra containing the sets of the form $A \times B$ where A is measurable in $[0, T]$ and B is Borel in E_n , then there exists a measurable selector of F .* See [4]. If the function F satisfies the condition 2 we say that F has a measurable graph. Another sufficient assumption is that the graph is analytic, see [3; Proposition 2.1]. We adopt the condition of the measurability of the graph since then the results will hold also when integrating over an abstract measurable space. On the real line we can change a function with a measurable graph on a set of measure zero and get a Borel-measurable graph. Obviously, such a change does not affect the integration.

The following notations will be used. The scalar product of the vectors p and x in E_n is denoted $p \cdot x$. The sum of the subsets A and B is $A + B = \{a + b: a \in A, b \in B\}$ and $\alpha A = \{\alpha a: a \in A\}$ where $\alpha > 0$ is a real number. We denote $\text{cl } A$ the closure of the set A . If A is convex, then $\text{ri } A$ is its relative interior.

2. Some preliminaries on the support function. Let A be a subset of E_n . The support function $s(p, A)$ of A is defined for all $p \in E_n$ by

$$s(p, A) = \sup \{p \cdot x: x \in A\}.$$

The following result can be found in [11, Theorem 13.2 and corollary 13.2.1].

Lemma 2.1. *The support function $s(\cdot, A)$ is a convex, lower semicontinuous and positively homogeneous function from E_n into $(-\infty, \infty]$. Conversely, if $s(p)$ is a convex function from E_n into $(-\infty, \infty]$ which is positively homogeneous and lower semicontinuous, then it is the support function of a certain nonempty closed and convex set A . This set A is unique and given by $A = \{x: p \cdot x \leq s(p) \text{ for all } p \in E_n\}$.*

Let F be a set-valued function. We denote $s(p, t) = s(p, F(t))$, i.e. $s(p, t)$ is the support function of the set $F(t)$. If F has a measurable graph then $s(p, \cdot)$ is Lebesgue measurable. The following result can be briefly stated as: *The support function of the integral is the integral of the support function.* It appears in the literature in several forms, see for instance [12; Theorem 4.1]. We state it here in a form suitable to us, and for completeness give a proof.

Lemma 2.2. *Let F be a set-valued function with a measurable graph. Suppose that for every p the integral $\int s(p, t) dt$ exists and is finite or $+\infty$. Then $s(p) = \int s(p, t) dt$ is the support function of a certain closed convex set A , and if $\int F$ is not empty, then $A = \text{cl } \int F$.*

Proof. Since for a fixed t the function $s(\cdot, t)$ is convex and positively homogeneous it follows that its integral $s(\cdot)$ is also convex and positively homogeneous.

We will show that $s(\cdot)$ is lower semicontinuous and this, together with Lemma 2.1, will assure the existence of A . Let s' denote the closure of s (see [11] for definition and properties of the closure of a convex function). Then s' is lower semicontinuous and $s'(p)$ may be different from $s(p)$ only on the relative boundary of $P = \{p: s(p) < \infty\}$. Let q be in the relative boundary of P and let $q_0 \in \text{ri } P$. Then s' is continuous on the segment $[q_0, q]$, and equal to s on $[q_0, q]$. The equality $s'(q) = s(q)$ will follow now from the continuity of s on $[q_0, q]$. We shall now prove this continuity. Since s is convex it follows that

$$s(q) \geq \limsup_{\substack{p \rightarrow q \\ p \in [q_0, q]}} s(p).$$

On the other hand $s(p, t)$ is continuous on $[q_0, q]$ for every t , and its convexity implies that $s(p, t) \geq \min \{s(q, t), 2s(q_1, t) - s(q_0, t)\}$ where $q_1 = (1/2)(q_0 + q)$ and $p \in [q_1, q]$. Thus, we can apply Fatou's Lemma and get

$$s(q) = \int s(q, t) = \int \lim s(p, t) \leq \liminf \int s(p, t) = \liminf s(p)$$

where the limits are taken for $p \rightarrow q$ and $p \in [q_0, q]$. The combination of the two inequalities gives the desired result.

Suppose now that $\int F$ is not empty, and let f be an integrable selector of F . Let $p \in E_n$ and let $\epsilon > 0$. Suppose $s(p, t)$ is Borel measurable (or change it on a null set in order to make it so) and denote

$$F_1(t) = \{x \in F(t): p \cdot x \geq s(p, t) - \epsilon\}$$

It is easily verified that F_1 has a measurable graph and thus it has a measurable selector f_1 . Define $g(t) = f_1(t)$ if $\|f_1(t)\| \leq M$ and $g(t) = f(t)$ otherwise. Then $g(t)$ is integrable and it is a selector of F . Also $p \cdot \int g(t) = \int p \cdot g(t)$ is close to $\int s(p, t)$ as ϵ is small and M is large enough. This proves that $s(p, \int F) \geq \int s(p, t)$. Since the inverse inequality is obvious this proves that $\int s(p, t)$ is the support function of $\text{cl } \int F$.

Remark. The nonemptiness of $\int F$ is necessary in the last lemma. Indeed let $F(t) = \{(x, y) \text{ in } E_2 : y = tx + t^{-1}\}$. Then $\int F = \emptyset$ but $A = E_2$.

3. The Bochner integral. The Hausdorff distance $\delta(A_1, A_2)$ between the two nonempty closed sets A_1 and A_2 in E_n is defined by

$$\delta(A_1, A_2) = \inf \{\lambda: A_1 + \lambda B \supset A_2, A_2 + \lambda B \supset A_1\},$$

where $B = \{x: \|x\| \leq 1\}$. We allow here a $+\infty$ distance. (A finite metric space can be obtained by taking the "arctg" of our distance.) Restriction to the closed and convex sets gives the following easy result

$$\delta(A_1, A_2) = \sup \{|s(p, A_1) - s(p, A_2)|: \|p\| = 1\}$$

where s is the support function and $\infty - \infty = 0$. Thus the subspace of the

closed convex sets can be isometrically embedded in the space of the support functions (see Lemma 2.1) with the metric

$$\delta(s_1, s_2) = \sup \{|s_1(p) - s_2(p)| : \|p\| = 1\}.$$

This property is widely known, see [7]; it goes back to Minkowski. The embedding enables us to derive easily some properties of the space of closed and convex subsets. For instance this space is complete. Indeed the pointwise limit of a Cauchy sequence of support functions is according to Lemma 2.1 a support function. The connected components of this space may be characterized as following. Given a set A (closed and convex) then the collection $\{B : \delta(A, B) < \infty\}$ is an arcwise connected component of A . Obviously it is an open set. A path between A and B where $\delta(A, B) < \infty$ is given in terms of the support functions by $s_\lambda(p) = \lambda s(p, A) + (1 - \lambda)s(p, B)$ for $0 \leq \lambda \leq 1$.

Another useful fact about the embedding is that it is linear in the sense $s(p, A + B) = s(p, A) + s(p, B)$ and $s(p, \alpha A) = \alpha s(p, A)$ if $\alpha > 0$. This will enable us to identify at least parts of the space as cones in a certain Banach space. A necessary condition for A and B to be members in a Banach space is $\delta(A, B) < \infty$. For this it is necessary that $P(A) = P(B)$ where $P(A) = \{p : s(p, A) < \infty\}$ is the support cone of A , and $P(B)$ is the support cone of B . The equality $P(A) = P(B)$ is not sufficient for $\delta(A, B) < \infty$. Indeed let $A = \{(x, y) \in E_2 : x \geq y^2\}$, then $P(A) = P((1/2)A)$ but $\delta(A, (1/2)A) = \infty$. In this case $P(A)$ is not a closed cone, but even closedness will not help. For instance consider in E_3 the cone generated by $D = \{(x, y, z) : z = 1, 0 \leq x \leq 1, |y| \leq x^{1/3}\}$ and the two support functions generated by $s_1(p) \equiv 0$ and $s_2(p) = y^2 x^{-1}$ on D .

The right object for the embedding of closed convex sets in a Banach space will be the collection of the sets with a fixed finitely generated support cone. The collection of the compact sets is an example. There the support cone is E_n^- . Another example which appears frequently is the collection of sets which are contained in a translation of the positive orthant, and contain a translation of it, i.e. the support cone is $E_n^- = \{x : x^i \leq 0 \ i = 1, \dots, n\}$.

In the following let P be a finitely generated cone (i.e., $P = \{\sum \lambda_i x_i : \lambda_i \geq 0\}$ with $x_1, \dots, x_m$ fixed), and let Γ be the space of those convex and closed sets A such that $P(A) = P$.

Lemma 3.1. *If $A \in \Gamma$, then $s(p, A)$ is continuous on P .*

Proof. Since P is finitely generated it follows that s , which is convex, is bounded on bounded subsets of P . Being a polyhedral set P has the property that for every $x \in P$ there is a positive η such that if $x + z \in P$ then $x + \eta(z\|z\|^{-1}) \in P$. Let $x \in P$ and let M be a bound of s for $\{y : \|x - y\| \leq \eta\}$. The convexity now implies

$$\frac{s(y) - s(x)}{\|y - x\|} \leq \frac{M - s(x)}{\eta}$$

if $y \in P$, $\|x - y\| \leq \eta$ and $y \neq x$. This implies the upper semicontinuity of the function s . Since lower semicontinuity is guaranteed it follows that $s(p, A)$ is continuous in P .

Theorem 3.2. *The mapping $A \rightarrow s(p, A)$ is an isomorphism and linear embedding of Γ into the Banach space of continuous functions on $\{p \in P: \|p\| = 1\}$ with the sup-norm. In this embedding Γ is a convex closed cone which generates the whole space.*

Proof. The first sentence is a summary of the discussion above. The last claim can be easily derived from the Stone-Weierstrass theorem.

Remark. Notice that the element which is mapped on the zero element is the asymptotic cone of an element in Γ . Thus an appropriate definition of $0 \cdot A$ will be the asymptotic cone of A , or $0 \cdot \infty = \infty$ in terms of the support function.

The following theorem partially overlaps Theorem 4.1 in [12].

Theorem 3.3. *Suppose that F is a set-valued function with a measurable graph and with values in Γ . If for every $p \in P$ the integral $\int s(p, t) dt$ is finite, then $\int F$ is nonempty, closed and $\int F \in \Gamma$.*

Proof. If $\int F \neq \emptyset$, then the integrability condition together with Lemma 2.2 imply that $\text{cl } \int F \in \Gamma$. Every set in Γ is of the form $B \times L^+$ where $B \subset L = P - P$ and L^+ is the space orthogonal to $L = P - P$ in E_n . In particular $F(t) = G(t) \times L^+$ for every t and $\int F = (\int G) \times L^+$. We now show that $\int G$ is nonempty and closed. Let p be in the relative interior of P . Define

$$G_1(t) = \{x \in G(t): p \cdot x \geq s(p, t) - 1\}.$$

Then G_1 has a measurable graph. Let $g(t)$ be a measurable selector of G_1 . Then $\int |p \cdot g(t)| < \infty$. Also if $q \in P - P$ is small enough then $\int (p + q) \cdot g(t) < \infty$, this because $p \in \text{ri } P$. This implies that g is integrable, and thus $\int G \neq \emptyset$. In order to show that $\int G = \text{cl } \int G$ it is enough to show that the extreme points and the extreme rays of $\text{cl } \int G$ belong to $\int G$. This because every closed convex set with no lines is the convex hull of its extreme points and extreme rays, and we know that $\int G$ is convex. A proof for the statement concerning the extreme points can be found in [2], see also [12; Theorem 4.3], the result is due to R. Borges. If $x + R$ is a ray in $\text{cl } \int G$ (where $R = \{\lambda y: \lambda \geq 0\}$ for a certain y) then $x(t) + R$ is a ray in $G(t)$ for every t and every $x(t) \in G(t)$. In particular if $\int x(t) = x$ it follows that $\int x(t) + R \supset x + R$. Apply this for an extreme ray in $\text{cl } \int G$ which is given by $x + R$ where x is an extreme point, and thus $x = \int x(t)$ for a certain integrable selector $x(t)$ of G .

Since $\int G$ is nonempty and closed it follows that $\int F$ is nonempty and closed. Now the first sentence of the proof can be reread as $\int F \in \Gamma$, which completes the proof.

Theorem 3.4. *Under the assumptions of Theorem 3.3 the function $t \rightarrow s(p, t)$ is Bochner integrable, and the integral is the support function of $\int F$. Conversely,*

if $t \rightarrow s(p, t)$ is Bochner integrable, then the integral is the support function of $\int F$ and the latter is nonempty and closed.

Proof. Let $\{p_k\}$, $k = 1, 2, \dots$ be a dense subset of $K = \{p \in P: \|p\| = 1\}$. Denote $B = \{f: \|f - f_0\| \leq \epsilon\}$ a ball in $C(K)$ the continuous functions on K with the sup-norm. We claim that the inverse image of B by $t \rightarrow s(\cdot, t) \in C(K)$ is measurable. Indeed

$$\{t: s(\cdot, t) \in B\} = \bigcap_{k=1}^{\infty} \{t: |s(p_k, t) - f_0(p_k)| \leq \epsilon\}.$$

Since $C(K)$ is separable it follows that the inverse image of any open set is measurable. Using once again the separability one can show that $t \rightarrow s(\cdot, t)$ is a limit of simple functions and thus strongly measurable, see [9, Definition 3.5.4]. Let $p_1, \dots, p_m$ be a finite set of generators of P , in K . Let $f(t)$ be an integrable selector of F , given by Theorem 3.3. The convexity of s and its definition i.e. $s(p, t) = \sup \{p \cdot x: x \in F(t)\}$ imply

$$- \|f(t)\| \leq s(p, t) \leq c \max \{|s(p_k, t)|: k = 1, \dots, m\}$$

with $c = \text{constant}$ and for every $p \in K$. Thus $\|s(\cdot, t)\|$ is integrable and hence (see [9; Theorem 3.7.4]) $t \rightarrow s(\cdot, t)$ is Bochner integrable. Once we know this, the integration can be carried out by computing $\int s(p, t)$ for every p , and together with Theorem 3.2 this completes the first part of the theorem. If $t \rightarrow s(\cdot, t)$ is Bochner integrable then $t \rightarrow s(p, t)$ is integrable for every $p \in P$, and again application of Theorem 3.3 completes the proof.

Remark. The last theorem says that under the assumptions of Theorem 3.3, the integration of the set-valued function and the Bochner integration of the vector-valued function after the embedding are equivalent.

As was noted in the introduction, Debreu [6] developed a similar theory for compact sets (in a Banach space). Also reference [10] deals with similar aspects. What our theorem adds (aside of the generalization to unbounded sets) is that the integrability of $\int s(p, t)$ for every p (indeed only the generators $p_1, \dots, p_m$ are needed) implies the Bochner integrability of F .

4. The indefinite integral. Recall that the indefinite integral of the set-valued function F is the set-valued function $U(t) = \int_0^t F(\tau) d\tau$.

This function is essentially (i.e. after an appropriate substitution) obtained as the attainable set of a linear control system. Thus an interesting case is when $U(t)$ is a compact set, and in the following we shall deal only with compact set-valued functions. Anyway, generalizations can be done simply by changing “compact” into “belongs to Γ ” where Γ is as described in the preceding Section.

Our first result will be to answer the question which serves as a title to Section 2 in [8], i.e. “when is a set-valued function an integral?” Our characterization is different than that in [8].

Following the classical treatment of real valued functions we say that a set-valued function $U(t)$ is absolutely continuous if for every $\epsilon > 0$ there exists $\eta > 0$ such that

$$\sum_{i=1}^k \delta(U(b_i), U(a_i)) < \epsilon$$

provided $a_1 < b_1 \leq a_2 < b_2 \leq \dots \leq a_k < b_k$ and $\sum (b_i - a_i) < \eta$. Here $\delta(\cdot, \cdot)$ is the Hausdorff distance, see Section 3. We shall write $\|A\| = \delta(A, \{0\})$. Notice that if U is an indefinite integral then $U(t)$ is convex for every t and $U(0) = \{0\}$.

Theorem 4.1. *Let U be a set-valued function on $[0, T]$, with convex and compact values, and such that $U(0) = \{0\}$. Necessary and sufficient conditions for U to be an indefinite integral are:*

- (a) *The function U is absolutely continuous.*
- (b) *If $t_1 > t_2$ in $[0, T]$, then there is a convex and compact set K such that $U(t_1) = U(t_2) + K$.*

Proof. Suppose $U(t) = \int_0^t F(\tau) d\tau$. Notice that if $b > a$ then $\delta(U(b), U(a)) = \|\int_a^b F(\tau) d\tau\|$. Notice also that

$$\left\| \int_a^b F \right\| = \sup_{\|p\|=1} \left| \int_a^b s(p, t) \right| \leq \sup_{\|p\|=1} \int_a^b |s(p, t)| \leq c \int_a^b \max_i |s(p_i, t)|$$

where $c = \text{constant}$, $1 \leq i \leq 2n$ and $p_1, \dots, p_{2n}$ are the $2n$ vectors $(0, \dots, 0, \pm 1, 0, \dots, 0)$. Since $\max_i |s(p_i, t)|$ is integrable it follows that for a given $\epsilon > 0$ there is an $\eta > 0$ such that $\int_c \max_i |s(p_i, t)|$ is less than ϵ provided the measure of C is less than η . Obviously the same η is an estimate for the validity of (a). Part (b) is necessary since

$$U(t_1) = \int_0^{t_1} F = \int_0^{t_2} F + \int_{t_2}^{t_1} F = U(t_2) + \int_{t_2}^{t_1} F.$$

The set $\int_{t_2}^{t_1} F$ is compact since both $U(t_2)$ and $U(t_1)$ are compact.

In order to show that (a) and (b) are sufficient we shall construct the support function of the desired F . Denote $S(p, t)$ the support function of U . Since $\delta(U(t), U(\tau)) \geq |S(p, t) - S(p, \tau)|$ if $\|p\| = 1$, and since U is absolutely continuous it follows that $S(p, \cdot)$ is absolutely continuous. Let $\{p_k\}$ be the rational vectors in E_n . Let $s(p_k, t)$ be the derivative of $S(p_k, t)$ with respect to t . We claim that $s(p_k, t)$ is almost everywhere convex and positively homogeneous on the rational vectors. That it is positively homogeneous is clear. The convexity follows from the fact that almost everywhere (and suppose that this a.e. is with respect to every p_k)

$$s(p_k, t) = \lim_{h \rightarrow 0+} \frac{1}{h} (S(p_k, t+h) - S(p_k, t))$$

Condition (b) implies that $S(p, t+h) - S(p, t)$ is a convex function (it is the support function of the guaranteed K) and thus $s(p_k, t)$ is convex as a

limit of convex functions. Since $s(p_k, t)$ is convex on the rationals one can use a slight modification of the classical proof for the continuity of convex functions (see [7; Theorem 4] and show that $s(p_k, t)$ is uniformly continuous on bounded sets. Let $s(p, t)$ be its extension to whole E_n . Thus $s(p, t)$ is *a.e.* convex and positively homogeneous. Also $s(p, t)$ is measurable in t for every p . Let $F(t)$ be the convex and compact set, such that $s(p, t) = s(p, F(t))$. On the rational vectors we have $\int_0^t s(p_k, \tau) d\tau = S(p_k, t)$. Thus the continuity and convexity imply $\int_0^t s(p, \tau) d\tau = S(p, t)$ for every p . Therefore for every t we have $\int_0^t F = U(t)$.

Remark 4.2. Notice that in the sufficiency case we did not use the full power of condition (a); we used only the absolute continuity of $S(p, t)$ for every p .

The following two theorems are generalizations of similar and known theorems in the theory of integration. They were proved by Hermes [8] for the case that the values are contained in a fixed bounded ball. Using the techniques of integration by means of the support function the proofs become simpler.

Theorem 4.3. *Let F be an integrably bounded set-valued function with closed values. Then almost everywhere*

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_t^{t+h} F(\tau) d\tau = \text{convex hull of } F(t).$$

Proof. In view of the real-valued case we have that *a.e.*

$$s(p, t) = \lim_{h \rightarrow 0} \frac{1}{h} \int_t^{t+h} s(p, \tau) d\tau.$$

Let us take this *a.e.* uniformly with respect to the rational vectors. This implies the desired equality for the support functions and it is equivalent to equality for the closed convex sets.

Theorem 4.4. *Let F_k be a sequence of compact set-valued functions, all are bounded by the same integrable function g . If the indefinite integrals $U_k = \int F_k$ converge uniformly (in the Hausdorff metric) to the set-valued function U , then U is an indefinite integral.*

Proof. Let s_k, S_k and S be the support functions of F_k, U_k and U respectively. Then for each p , where $\|p\| = 1$, the function $s_k(p, t)$ is bounded by $g(t)$ and $S_k(p, t) \rightarrow S(p, t)$ uniformly. This implies the real value function $S(p, \cdot)$ is absolutely continuous. Obviously condition (b) of Theorem 4.1 is valid. In view of Theorem 4.1 and Remark 4.2 the set-valued function $U(t)$ is an indefinite integral.

Acknowledgement. The author wants to thank the referee for some helpful comments and references. The author also thanks Y. Benyamini and Y. Kannai for a useful discussion.

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Date communicated: NOVEMBER 8, 1973