On p -Sidon Sets

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Introduction. Let G be a compact abelian group with dual group Γ . A subset E of Γ is p -Sidon ($1 \leq p < 2$) if there is a constant α such that each ϕ in $C(G)$ with $\hat{\phi}$ supported on E satisfies $\|\hat{\phi}\|_p \leq \alpha \|\phi\|_\infty$. Hence a set is 1-Sidon if and only if it is Sidon. Moreover a duality argument yields that E is p -Sidon if and only if $\ell^{p'}(E) \subset M(G)^\wedge|_E$, where the latter symbol denotes the restrictions of the Fourier-Stieltjes transforms to E and where $p' = p/(p - 1)$. Several of the basic results on p -Sidon sets were independently obtained by Bożejko and Pytlik [1], L.-S. Hahn [4], and Edwards and Ross [3]. The article of Edwards and Ross appears to contain all that was known about p -Sidon sets prior to this paper. Here we prove:

Theorem. *Suppose $A_1, \dots, A_n$ are mutually disjoint, infinite subsets of Γ whose union is dissociate. Then $E \equiv A_1 + \dots + A_n$ is p -Sidon if and only if $p \geq 2n/(n + 1)$.*

For $n = 2$ this theorem is due to Edwards and Ross [3, Corollary 5.5] and is one of the main contributions of their paper.

Let α_p denote the class of p -Sidon sets, for $1 \leq p < 2$. Suppose $1 \leq p_1 < p_2 < 2$. Clearly $\alpha_{p_1} \subset \alpha_{p_2}$. The question as to whether or not the containment is proper is certainly one of the fundamental questions of the theory. The result of Edwards and Ross implies that if $1 \leq p_1 < 4/3 \leq p_2 < 2$, then α_{p_1} is properly contained in α_{p_2} .

Perhaps most importantly it shows that there exists p ($1 < p < 2$) such that α_p is distinct from the class α_1 of ordinary Sidon sets. Our extension to arbitrary $n \geq 2$ of the Edwards-Ross result implies that an infinite number of the collections α_p are distinct. The general question of whether α_{p_1} is always properly contained in α_{p_2} for $1 \leq p_1 < p_2 < 2$ remains unresolved.

Our proof of the theorem above is divided into two sections. In the first section we begin by proving (Lemma 1) that the sum of any n infinite sets is never p -Sidon for $p < 2n/(n + 1)$. The lemma is a fairly direct extension of Corollary 2.7 of [3], but its proof does offer some additional technical difficulties and so we include it. One direction of our theorem follows immediately from

Lemma 1. In the rest of Section 1 we outline the proof of the converse. The outline follows the Edwards–Ross argument for $n = 2$ except for two key inequalities. These are proved in the second section and may be of independent interest. The first (Lemma 2) deals with the Varopoulos bimeasure norm estimates [10, pp. 82–85] and actually can be obtained from some of his estimates, but is stronger than his inequality 6.2.10 [10, p. 85]. Our proof, which is based on Rademacher series, seems more transparent. Lemma 3 extends to higher dimensions a mixed ℓ^p -norm inequality which is due to Littlewood [7, (7), p. 168].

We use the standard notation of Fourier analysis (see for instance [8]) which basically agrees with that of [3] except here $+$ denotes the group operation.

1. The argument. Denote by $|A|$ the cardinality of a set $A \subset \Gamma$. Let n be from here on a fixed positive integer. A pair of numbers $r, r' \geq 1$ will always be related by $r' = r/(r - 1)$.

Lemma 1. Suppose $E \subset \Gamma$ is p -Sidon for some $1 \leq p < 2n/(n + 1)$. Then $\sup \{ \min (|A_1|, \dots, |A_n|) : A_1 + \dots + A_n \subset E; A_1, \dots, A_n \subset \Gamma \} < \infty$.

Proof. Assume the lemma false and let $k \geq 2$ be a fixed integer. Then there exists $A_1, \dots, A_n \subset \Gamma$ with $|A_i| > 3^{kn-1}$ for $1 \leq j \leq n$ such that $A_1 + \dots + A_n \subset E$. Let $\gamma_1 \neq 0$ be in A_1 . We inductively pick k elements each from $A_1, \dots, A_n$ as follows. Suppose $1 \leq s < kn$ and $\gamma_1, \dots, \gamma_s$ satisfy

$$(1) \quad \gamma_\ell \in A_i \quad \text{if} \quad (j-1)k < \ell \leq jk \quad \text{and}$$

and

$$(2) \quad \gamma_\ell \notin B_{\ell-1} \equiv \left\{ \sum_{i=1}^{\ell-1} \delta_i \gamma_i : (\delta_1, \dots, \delta_{\ell-1}) \in \{-1, 0, 1\}^{\ell-1} \right\}.$$

Then there is a γ_{s+1} satisfying (1) and (2) since $|B_s| \leq 3^s < |A_i|$ for $1 \leq j \leq n$. It follows from (2) that for $(\delta_1, \dots, \delta_{kn}) \in \{-1, 0, 1\}^{kn}$

$$(3) \quad \sum_{i=1}^{kn} \delta_i \gamma_i = 0 \quad \text{if and only if} \quad \delta_1 = \delta_2 = \dots = \delta_{kn} = 0;$$

in particular note that if $i \neq j$, then $\gamma_i \notin \{0, \gamma_j, -\gamma_j\}$. Moreover let $C_i \equiv \{\gamma_\ell : (j-1)k < \ell \leq jk\} \subset A_i$. We claim $|C_1 + \dots + C_n| = k^n$. To be otherwise there must be points $(a_1, \dots, a_n) \neq (b_1, \dots, b_n)$ in $C_1 \times \dots \times C_n$ such that

$$0 = \sum_1^n a_i - \sum_1^n b_i = \sum_1^{kn} \delta_i \gamma_i$$

for some $(\delta_1, \dots, \delta_{kn}) \neq 0$ in $\{-1, 0, 1\}^{kn}$, which is impossible.

Consider now the Riesz polynomial

$$R(x) = \prod_{i=1}^{kn} [1 + \frac{1}{2}(\gamma_i(x) + \overline{\gamma_i(x)})]$$

and the characteristic function $\hat{g}$ of $C \equiv C_1 + \cdots + C_n \subset E$. Each factor of $R(x)$ is nonnegative and each term in the formal expansion of R has the form $2^{-c} \sum \delta_i \gamma_i$ where $c = \sum |\delta_i|$ for each choice of $(\delta_1, \dots, \delta_{kn}) \in \{-1, 0, 1\}^{kn}$. In particular $R \geq 0$, $\hat{R}(0) = 1$, $\hat{R} \geq 0$, and $R(0) = 2^{kn}$; hence $\|R\|_1 = 1$ and $\|R\|_2 \leq \|R\|_\infty = 2^{kn}$. Moreover, since $|C| = k^n$, we have

$$\|R * g\|_\infty \geq |R * g(0)| = \sum_{\gamma \in \Gamma} \hat{R}(\gamma) \hat{g}(\gamma) \geq 2^{-n} k^n.$$

This together with Theorem 2.4 of [3] and the inequality

$$\|R\|_r \leq \|R\|_1^{(2-r)/r} \|R\|_2^{2/r'} \leq \|R\|_\infty^{2/r'}$$

(see for instance [5, Theorem 13.19]) yields

$$2^{-n} k^n \leq \|R * g\|_\infty \leq \|R\|_r \|g\|_{r'} \leq 2^{2kn/r'} \alpha(r')^{1/2} \|\hat{g}\|_a,$$

where α depends only on E and $a = 2p/(3p - 2)$. Set $r' = k$. It follows that

$$2^{-n} k^n \leq \alpha 2^{2n} k^{1/2} (k^n)^{1/a}.$$

Since this must hold for any k , we conclude $p \geq 2n/(n + 1)$ and the proof is complete.

We point out that Lemma 1 is a fairly direct consequence of Theorems 4 and 5 in [6, pp. 57–8] when G is the circle group $[0, 2\pi)$.

This lemma established our theorem in one direction. To prove the other, we use the argument in [3, sections 4 and 5] which is given there for $n = 2$. With the exception of inequalities (5) and (6) to follow, that argument extends immediately to the general n and the extension is not technically difficult; hence we only summarize it here. The two inequalities are proved in the next section.

Suppose $A_1, \dots, A_n \subset \Gamma$ are mutually disjoint and infinite and that $\mathcal{Q} \equiv A_1 \cup \cdots \cup A_n$ is dissociate. That is, if $\gamma_1, \dots, \gamma_k$ are in $\mathcal{Q}$ and if $\delta_1, \dots, \delta_k$ are in $\{-2, -1, 0, 1, 2\}$, then $\sum \delta_i \gamma_i = 0$ implies $\delta_1 = \cdots = \delta_k = 0$. Set $A = A_1 \times \cdots \times A_n$ and $E = A_1 + \cdots + A_n$. It follows that the map $\rho : A \rightarrow E$ given by $\rho(\gamma_1, \dots, \gamma_n) = \gamma_1 + \cdots + \gamma_n$ is bijective. Let $B_1 \subset A_1, \dots, B_n \subset A_n$ be any fixed finite subsets and set $B = B_1 \times \cdots \times B_n$. Given real-valued functions ϕ_i in $\ell^\infty(B_i)$ with $\|\phi_i\|_\infty \leq 1$ ($1 \leq i \leq n$), consider the Riesz polynomial

$$R(x) = \prod_{i=1}^n \prod_{\gamma \in B_i} [1 + \frac{1}{2} \phi_i(\gamma)(\gamma(x) + \overline{\gamma(x)})].$$

The properties of R are similar to those discussed in Lemma 1 where each $\phi_i \equiv 1$. In particular $\|R\|_1 = 1$ and, since $\mathcal{Q}$ is dissociate, $\hat{R}(\rho(\gamma_1, \dots, \gamma_n)) = 2^{-n} \phi_1(\gamma_1) \cdots \phi_n(\gamma_n)$ on B . Thus the measure $\mu = 2^n R dx$ in $M(G)$ satisfies (i) $\hat{\mu}(\rho(\gamma_1, \dots, \gamma_n)) = \phi_1(\gamma_1) \cdots \phi_n(\gamma_n)$ on B and (ii) $\|\mu\| = 2^n$, where $\|\cdot\|$ denotes the total variation norm. If the ϕ_i in $\ell^\infty(B_i)$ are arbitrary, then by considering the real and imaginary parts separately of $\phi_i/\|\phi_i\|_\infty$, one sees that there is a

μ in $M(G)$ satisfying (i) with $\|\mu\| \leq 4^n \|\phi_1\|_\infty \cdots \|\phi_n\|_\infty$. It therefore follows that for each ϕ in $V \equiv \ell^\infty(B_1) \otimes \cdots \otimes \ell^\infty(B_n)$ there is a μ in $M(G)$ such that

$$(4) \quad \hat{\mu}(\rho(\gamma_1, \dots, \gamma_n)) = \phi(\gamma_1, \dots, \gamma_n) \text{ on } B$$

and

$$\|\mu\| \leq 5^n \|\phi\|_V,$$

where $\|\cdot\|_V$ denotes the projective $\otimes_\Pi$ norm. (Actually 5^n can be replaced by 4^n .) The necessary basic properties of V and its dual are discussed in the next section. For a complete discussion, see [10].

The topological dual V' of V can be realized algebraically as the vector space $\ell^\infty(B)$. The dual norm on V' is denoted by $\|\cdot\|_{BM}$. Let $(B)_i$ denote the set $B_1 \times \cdots \times B_{i-1} \times B_{i+1} \times \cdots \times B_n$. We prove in the next section that for ϕ in $\ell^\infty(B)$

$$(5) \quad \|\phi\|_{BM} \geq 8^{1-n} \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2}, \quad 1 \leq i \leq n,$$

and

$$(6) \quad \sum_{i=1}^n \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \geq \|\phi\|_V, \quad p = 2n/(n+1).$$

Set $\alpha = 8^{1-n}$. It follows that $\|\phi\|_{BM} \geq n^{-1}\alpha \|\phi\|_p$ and from a duality argument that

$$(7) \quad \|\phi\|_V \leq \alpha^{-1}n \|\phi\|_{p'}.$$

for every choice of $B_1, \dots, B_n$ and for all ϕ in $\ell^\infty(B)$.

Let ψ be in $\ell^{p'}(E)$ and set $\phi = \psi \circ \rho$. Then ϕ is in $\ell^{p'}(A)$ and $\|\psi\|_{p'} = \|\phi\|_{p'}$. Moreover (7) and (4) imply that there is a μ_B in $M(G)$ such that $\hat{\mu}_B = \psi$ on $B_1 + \cdots + B_n$ and $\|\mu_B\| \leq 5^n \alpha^{-1}n \|\psi\|_{p'}$. A weak* compactness argument therefore yields μ in $M(G)$ with $\|\mu\| \leq 5^n \alpha^{-1}n \|\psi\|_{p'}$ such that $\hat{\mu} = \psi$ on E . Thus $\ell^{p'}(E) \subset \hat{M(G)}|_E$. As noted in the introduction, this implies that E is a p -Sidon set.

2. The inequalities. In what follows n is a fixed positive integer, $B_1, \dots, B_n$ are finite sets, $B = B_1 \times \cdots \times B_n$, and $(B)_i = B_1 \times \cdots \times B_{i-1} \times B_{i+1} \times \cdots \times B_n$. Set $V = \ell^\infty(B_1) \otimes \cdots \otimes \ell^\infty(B_n)$ under the projective $\otimes_\pi$ norm which is denoted by $\|\cdot\|_V$ and let V' be its topological dual with dual norm denoted by $\|\cdot\|_{BM}$. Recall that since B is finite both V and V' can be realized algebraically as the vector space $\ell^\infty(B)$ and we freely do this. The norm on V is given by

$$\|\phi\|_V = \inf \sum_i \|\phi_1^{(i)}\|_\infty \cdots \|\phi_n^{(i)}\|_\infty$$

where the infimum is taken over all finite sum representations $\sum_i \phi_1^{(i)} \cdots \phi_n^{(i)}$ of ϕ . That is, $\phi_j^{(i)}$ is in $\ell^\infty(B_j)$ for $1 \leq j \leq n$ and $\phi(\gamma_1, \dots, \gamma_n) =$

$\sum_i \phi_1^{(i)}(\gamma_1) \cdots \phi_n^{(i)}(\gamma_n)$ on B . Also, in this setting, the value of a μ in V' at a basic element $\phi_1 \cdots \phi_n$ in V is given by

$$\mu(\phi_1 \cdots \phi_n) = \sum_B \mu(\gamma_1, \cdots, \gamma_n) \phi_1(\gamma_1) \cdots \phi_n(\gamma_n).$$

This completely determines μ on V . Since the unit ball of V is the closed convex hull of

$$S \equiv \{\phi_1 \cdots \phi_n : \phi_i \in \ell^\infty(B_i), \|\phi_i\|_\infty \leq 1, 1 \leq j \leq n\},$$

the V' norm of μ is $\|\mu\|_{BM} = \sup \{|\mu(\phi)| : \phi \in S\}$. By summing first over $(B)_i$ and then over B_i one verifies that for $1 \leq i \leq n$

$$\begin{aligned} \|\mu\|_{BM} &= \sup \left\{ \left| \sum_{B_i} \phi_i(\gamma_i) \sum_{(B)_i} \mu(\gamma_1, \cdots, \gamma_n) \phi \right| : \phi \in (S)_i, \phi_i \in \ell^\infty(B_i), \|\phi_i\|_\infty \leq 1 \right\} \\ &\geq \sup \left\{ \sum_{B_i} \left| \sum_{(B)_i} \mu(\gamma_1, \cdots, \gamma_n) \phi \right| : \phi \in (S)_i \right\}, \end{aligned}$$

where

$$(S)_i = \{\phi_1 \cdots \phi_{i-1} \cdot \phi_{i+1} \cdots \phi_n : \phi_j \in \ell^\infty(B_j), \|\phi_j\|_\infty \leq 1, 1 \leq j \neq i \leq n\}.$$

It should be noted again that the preceding information on tensor products and much more can all be found in [10, section 2]. We have only summarized the essential facts using somewhat different notation. Let us now prove inequality (5).

Lemma 2. *Let $1 \leq i \leq n$ and suppose μ is in $\ell^\infty(B)$. Then*

$$\|\mu\|_{BM} \geq 8^{1-n} \sum_{B_i} \left(\sum_{(B)_i} |\mu(\gamma_1, \cdots, \gamma_n)|^2 \right)^{1/2}.$$

Proof. Number the elements in each B_i from 1 to $|B_i|$. If γ is the k^{th} element of B_i let $r_\gamma(t) = \text{sgn} [\sin (2^k t \pi)]$ be the k^{th} Rademacher function where t is in $[0, 1]$, $t \neq \ell 2^{-k}$, $\ell = 0, 1, \cdots, 2^k$; on these exceptional points set $r_\gamma(t) = 1$. It is well known [2, Theorem 14.2.1] that as functions on $[0, 1]$ the Rademacher series satisfy

$$\left\| \sum_{B_i} c_\gamma r_\gamma \right\|_1 \geq 8^{-1} \left(\sum_{B_i} |c_\gamma|^2 \right)^{1/2}.$$

It follows from this and Minkowski's inequality for integrals that as functions on $[0, 1]^n$ the multivariable Rademacher series satisfy

$$\left\| \sum_B \psi(\gamma) r_\gamma \right\|_1 \geq 8^{-n} \left(\sum_B |\psi(\gamma)|^2 \right)^{1/2}$$

for any ψ in $\ell^\infty(B)$ where $\gamma = (\gamma_1, \cdots, \gamma_n)$ is in B and

$$r_\gamma(t_1, \cdots, t_n) = r_{\gamma_1}(t_1) \cdots r_{\gamma_n}(t_n).$$

(See [9, pp. 277–8].)

Given γ in B denote by $(\gamma)_i$ in $(B)_i$ the element obtained from γ by deleting the i^{th} component. The earlier estimate for $\|\mu\|_{BM}$ together with the preceding inequalities applied to $n - 1$ variables yields

$$\begin{aligned}
\|\mu\|_{BM} &\geq \sup_{[0,1]^{n-1}} \sum_{B_i} \left| \sum_{(B)_i} \mu(\gamma) r_{(\gamma)_i} \right| \\
&\geq \int_{[0,1]^{n-1}} \sum_{B_i} \left| \sum_{(B)_i} \mu(\gamma) r_{(\gamma)_i}(t) \right| dt \\
&= \sum_{B_i} \left\| \sum_{(B)_i} \mu(\gamma) r_{(\gamma)_i} \right\|_1 \\
&\geq 8^{1-n} \sum_{B_i} \left(\sum_{(B)_i} |\mu(\gamma)|^2 \right)^{1/2}.
\end{aligned}$$

This completes the proof.

The next lemma establishes inequality (3). We note that it extends immediately to infinite B_i 's.

Lemma 3. *Let $p = 2n/(n+1)$ and suppose ϕ is in $\ell^\infty(B)$. Then*

$$\sum_{i=1}^n \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \geq \|\phi\|_p.$$

Proof. For each $1 \leq i \leq n$, B_i can be linearly ordered so that the finite sequence

$$\left\{ \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \right\}_{\gamma_i \in B_i}$$

is nonincreasing. Fix such an ordering and denote it by " $\leq$ ". Given γ_i in B_i denote by $|\gamma_i|$ the cardinality of $\{\rho_i \in B_i : \rho_i \leq \gamma_i\}$. For each positive integer k and each i let

$$(B)_{i,k} = \{(\gamma_1, \dots, \gamma_{i-1}, \gamma_{i+1}, \dots, \gamma_n) \in (B)_i : |\gamma_i| \leq k, 1 \leq j \neq i \leq n\}.$$

Let i and γ_i in B_i be fixed. Hölder's inequality yields

$$\begin{aligned}
\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^p &\leq |\gamma_i|^{(n-1)/(n+1)} \left(\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^{p(n+1)/n} \right)^{n/(n+1)} \\
&= |\gamma_i|^{(n-1)/(n+1)} \left(\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{(n-1)/2(n+1)} \\
&\quad \cdot \left(\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2}.
\end{aligned}$$

In addition, the nonincreasing condition implies

$$\begin{aligned}
|\gamma_i| \left(\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} &\leq |\gamma_i| \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \\
&\leq \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2},
\end{aligned}$$

where in the first two expressions immediately above γ_i in B_i should still be considered as fixed. It now follows that

$$\begin{aligned}
\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^p &\leq \left[\sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \right]^{(n-1)/(n+1)} \left(\sum_{(B)_{i,|\gamma_i|}} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2}.
\end{aligned}$$

Since this holds for any γ_i in B_i , we have

$$\begin{aligned} S_i &\equiv \sum_{\gamma_i \in B_i} \sum_{(B)_i, |\gamma_i|} |\phi(\gamma_1, \dots, \gamma_n)|^p \\ &\leq \left[\sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \right]^{(n-1)/(n+1)} \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \\ &= \left[\sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2} \right]^p \end{aligned}$$

Since the ℓ^1 -norm dominates the ℓ^p -norm,

$$\left(\sum_{i=1}^n S_i \right)^{1/p} \leq \sum_{i=1}^n \sum_{B_i} \left(\sum_{(B)_i} |\phi(\gamma_1, \dots, \gamma_n)|^2 \right)^{1/2}.$$

Finally observe that

$$\left(\sum_B |\phi|^p \right)^{1/p} \leq \left(\sum_{i=1}^n S_i \right)^{1/p}.$$

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