Observation Operators and Cosmology

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1. Introduction. Because the resolving power of any optical or radio telescope has a definite lower bound, the fundamental observables in distant galaxies derive from aggregate properties of stars, gasses, and dust clouds rather than from the individual properties of these constituents. Thus, if theory and observation are to mesh, models of galactic structures should be representative of aggregate or averaged behavior; indeed, it is only through considerations of such averages that mass density and luminosity are meaningfully associated with galaxies. As the domains of general relativity and classical relativistic cosmology are currently envisioned, galaxies and clusters of galaxies play the role of indicator points whose properties must themselves be averaged in order to determine the salient parameters of the various cosmological models. Unfortunately, classical cosmology denies the very existence of such indicator points and hence a description of the averaging processes and domains of averaging that are consistent with the classical models can not be obtained within the discipline of classical cosmology.

On the other hand, the Einstein field equations are experimentally known to relate only the local differential geometric quantities (curvature and infinitesimal metric structure) and the local momentum-energy distributions that are characteristic of a stellar or planetary neighborhood. Since these field equations and their associated physical interpretations are carried over directly as a basis for the classical cosmological models and for the description of galaxies and clusters of galaxies, a fundamental problem must be resolved—namely, the construction of the governing equations for aggregates when the individual elements of the aggregate are governed by the Einstein field equations. A solution of this problem will give not only a clear understanding of the gross structures of the universe in terms of the known properties of their constituents, it will also define the possible structural environments for galaxies and clusters of galaxies that is demanded by the conditions of internal consistency with the Einstein field equations.

This paper is dedicated to my friend and mentor, Tracy Yerkes Thomas, on the occasion of his 70th birthday.

2. Observation operators. We start with a 4-dimensional metric space $\mathcal{E}$ of the hyperbolic-normal type, whence the signature of $\mathcal{E}$ is -2 . Let $\mathbf{0}$ denote a generic point of $\mathcal{E}$ which we identify with the observer at the instant of observation. Such an observer erects a tetrad of linearly independent contravariant vectors $\{W_a^A(\mathbf{0})\}$ at $\mathbf{0}$ by means of which he can reduce any linear homogeneous geometric object (vectors, tensors, etc.) at the point $\mathbf{0}$ to an ordered array of scalars by the process of transvection. Capital Latin indices will be used throughout this paper for tensor indices while lower case Latin indices are used as labeling indices. Both kinds of indices are assumed to obey the summation convention with the range 0, 1, 2, 3. Thus, if $\{T_A(\mathbf{0})\}$ is a covariant vector field at $\mathbf{0}$, the corresponding scalars are given by

$$(2.1) \quad T_a(\mathbf{0}) = T_A(\mathbf{0})W_a^A(\mathbf{0}).$$

Physically, the scalars $\{T_a(\mathbf{0})\}$ may be viewed as the values of the field $\{T_A(\mathbf{0})\}$ when this field is resolved on the "laboratory" or "reference" frame that is defined by the tetrad $\{W_a^A(\mathbf{0})\}$.

Commencing with the tetrad $\{W_a^A(\mathbf{0})\}$, we can extend it to a field of tetrads $\{W_a^A(\mathbf{x})\}$ over $\mathcal{E}$ in any number of different ways. For instance, $\{W_a^A(\mathbf{x})\}$ can be defined by parallel transport of $\{W_a^A(\mathbf{0})\}$ along the geodesic in $\mathcal{E}$ that connects the points $\mathbf{0}$ and $\mathbf{x}$, provided $\mathbf{x}$ lies in the neighborhood of $\mathbf{0}$ for which the geodesic connectivity is simple. A possibly more useful method is to take $\{W_a^A(\mathbf{0})\}$ as a linearly independent system of (two complex conjugate and two real) null vectors at $\mathbf{0}$ and then to imbed $\{W_a^A(\mathbf{0})\}$ in a normed null basis [1, 2, 3] on $\mathcal{E}$. If this is done, the tetrad forms what may be referred to as a "radiation frame" since two of its constituent vectors are real null vectors that are tangent to the null cone at each point of $\mathcal{E}$. Such a structure is certainly a natural one in dealing with astronomical bodies since the radiation frame is tailor-made for representing information that is propagated over the null cone.

Regardless of the manner in which $\{W_a^A(\mathbf{0})\}$ is extended to a system of tetrad fields on $\mathcal{E}$, other than the obvious smoothness conditions and the requirement

$$(2.2) \quad \det [W_a^A(\mathbf{x})] \neq 0$$

for all $\mathbf{x}$ in $\mathcal{E}$, a reference frame is defined at each point of $\mathcal{E}$ by the tetrad field. In view of the requirement (2.2), there exists a unique inverse (dual) system of fields $\{W_A^a(\mathbf{x})\}$ defined by

$$(2.3) \quad W_A^a(\mathbf{x})W_a^A(\mathbf{x}) = \delta_b^a, \quad W_A^a(\mathbf{x})W_a^B(\mathbf{x}) = \delta_A^B.$$

These relations motivate the reference to a tetrad field and its dual as the intermediary components of the identity [4]. If the fields $\{W_A^a(\mathbf{x})\}$ happen to be such that

$$(2.4) \quad \nabla_{[A}W_{B]}^b = 0,$$

the covariant tetrad is said to form a holonomic frame [5]. If, on the other hand, (2.4) is not satisfied, the tetrad is said to form an anholonomic frame.

The term “anholonomic coordinates” is prevalent in the literature and arose principally due to the foibles of history. It is really the frame that is holonomic or anholonomic and this has nothing to do with coordinates or coordinate coverings; anholonomic coordinates are almost a contradiction in terms since they give a pseudo-coordinatization of a region of a manifold that is path dependent and arises from pathwise integration of nonintegrable Pfaffians, as opposed to the natural coordinatization that results from the integration of completely integrable Pfaffians. Anholonomic coordinates arose because of the convenience afforded in interpreting anholonomic frames of reference in terms of the differential of a coordinate transformation at a point. The problem with anholonomic coordinates comes in the attempt to extend the coordinate differential interpretation to a finite neighborhood of a point.

It is probably evident to the reader that most frames of reference used in physics are anholonomic frames since they are chosen primarily for the convenience of the experimentalist rather than from their natural adaptability to an underlying geometry in the global sense. We consequently assume that the frames defined by the tetrad under consideration is anholonomic since this assumption results in no loss of generality.

Any linear geometric object field at any point of $\mathcal{E}$ can be reduced to an ordered array of scalar functions by transvection with the appropriate members of the anholonomic frame and its dual at the point in question. For example,

$$t_{ab}^c(\mathbf{x}) = T_{AB}^C(\mathbf{x})W_c^A(\mathbf{x})W_b^B(\mathbf{x}).$$

Such a collection of scalar functions represents the results that would be obtained by an observer at the point $\mathbf{x}$ in $\mathcal{E}$ by the resolution of the field $\{T_{AB}^C(\mathbf{x})\}$ with respect to the reference frame defined by $\{W_a^A(\mathbf{x})\}$. Mathematically, the scalars $\{t_{ab}^c(\mathbf{x})\}$ are referred to as the anholonomic components of the field $\{T_{AB}^C(\mathbf{x})\}$ with respect to $\{W_a^A(\mathbf{x})\}$ and depend in an intrinsic fashion on the choice of the reference frame. It is accordingly obvious that the relations between anholonomic components of fields at different points involve not only the changes in the field values from point to point, but also on the manner in which the anholonomic reference frame varies from point to point. This dependence on the anholonomic reference frame may be viewed as the inseparable dependence of observed quantities on the processes and techniques of observation.

Let us identify ourselves with the observer at $\mathbf{0}$, and ask what we may infer by observation as to the state of affairs at a generic point $\mathbf{x}$ of $\mathcal{E}$. For simplicity, let us assume that we are concerned with a covariant vector field $\{T_A(\mathbf{x})\}$. If $\mathbf{y}$ denotes a generic point of $\mathcal{E}$, an observer at $\mathbf{y}$ would record, with respect to his reference frame $\{W_a^A(\mathbf{y})\}$, the scalars

$$(2.5) \quad t_a(\mathbf{y}) = T_A(\mathbf{y})W_a^A(\mathbf{y}).$$

If there were no error in the observations and if there were a perfect communication process between $\mathbf{y}$ and $\mathbf{0}$, the observer at $\mathbf{0}$ could claim that the field

scalars he would "see" at $\mathbf{x}$ would be given by $\{t_a(\mathbf{x})\}$. [In effect, this amounts to the assumption that the observer at $\mathbf{0}$ can follow his reference frame as it evolves on passage from $\mathbf{0}$ to $\mathbf{y}$. This ability is what is hidden in the assumption that there is a perfect communication process between $\mathbf{y}$ and $\mathbf{0}$.] It then follows, on the assumption that the observer at $\mathbf{0}$ knows the dual frame $\{W_B^b(\mathbf{y})\}$, that a transvection (on lower case Latin indices) of (2.5) with $\{W_B^b(\mathbf{y})\}$ reproduces the field $\{T_B(\mathbf{y})\}$.

In actual fact, any observation process always involves certain intrinsic errors and imperfect communication processes. In addition, it is quite often necessary to restrict the investigation to certain ensemble properties or averages due to the inability of the observer to control or affect the observation process or its environment. Under these circumstances, the field of scalars that an observer at $\mathbf{0}$ ascribes to a point $\mathbf{x}$ would be a composite of contributions from field values at points neighboring to $\mathbf{x}$. This has the effect of converting a point-valued field into a set-valued field.

An adequate description of the observation process and its intrinsic limitations requires the introduction of certain additional quantities. Let $M_c^b(\mathbf{y}, \mathbf{x}; \mathbf{0})$ denote the contribution to the c^{th} scalar that an observer at $\mathbf{0}$ would ascribe to the point $\mathbf{x}$ due to the unit b^{th} scalar at the point $\mathbf{y}$ per unit of invariant (volume) measure $dv(\mathbf{y})$ of $\mathcal{E}$ at the point $\mathbf{y}$. In most cases, it is sufficient to take $dv(\mathbf{y})$ to be the proper infinitesimal volume element of $\mathcal{E}$ at $\mathbf{y}$ since any singularities of the observation process may be accounted for by an appropriate restriction of the support of the scalar functions M_c^b or by the extension of these functions to generalized scalar functions in the sense of distributions. It then follows that the composite scalar fields that an observer at $\mathbf{0}$ should ascribe to the point $\mathbf{x}$ under these circumstances would be given by

$$(2.6) \quad t_c(\mathbf{x}; \mathbf{0}) = \int t_b(\mathbf{y}) M_c^b(\mathbf{y}, \mathbf{x}; \mathbf{0}) dv(\mathbf{y}).$$

When this is transvected on lower case Latin indices with $\{W_B^c(x)\}$, we obtain the vector field

$$(2.7) \quad \langle T_B(\mathbf{x}) \rangle = W_B^c(\mathbf{x}) \int t_b(\mathbf{y}) M_c^b(\mathbf{y}, \mathbf{x}; \mathbf{0}) dv(\mathbf{y})$$

that an observer at $\mathbf{0}$ would ascribe to the point $\mathbf{x}$. That $T_B(\mathbf{x})$ always constitutes the components of a vector field follows from previously published results [6].

If we combine the above equations, we obtain

$$(2.8) \quad \langle T_B(\mathbf{x}) \rangle = \int \mathcal{I}C_B^A(\mathbf{y}, \mathbf{x}; \mathbf{0}) T_A(\mathbf{y}) dv(\mathbf{y})$$

where

$$(2.9) \quad \mathcal{I}C_B^A(\mathbf{y}, \mathbf{x}; \mathbf{0}) = W_a^A(\mathbf{y}) M_b^a(\mathbf{y}, \mathbf{x}; \mathbf{0}) W_B^b(\mathbf{x}).$$

On the other hand, if we had started with a contravariant vector field $\{T^A(\mathbf{y})\}$,

then (2.6) would have been replaced by

$$(2.10) \quad t^b(\mathbf{x}; \mathbf{0}) = \int t^a(\mathbf{y}) N_a^b(\mathbf{y}, \mathbf{x}; \mathbf{0}) dv(\mathbf{y}),$$

where N_a^b plays the same role for contravariant quantities as M_a^b plays for covariant quantities. Accordingly, we obtain

$$(2.11) \quad \langle T^B(\mathbf{x}) \rangle = \int \mathcal{L}_A^B(\mathbf{y}, \mathbf{x}; \mathbf{0}) T^A(\mathbf{y}) dv(\mathbf{y})$$

where

$$(2.12) \quad \mathcal{L}_A^B(\mathbf{y}, \mathbf{x}; \mathbf{0}) = W_A^a(\mathbf{y}) N_a^b(\mathbf{y}, \mathbf{x}; \mathbf{0}) W_b^B(\mathbf{x}).$$

The operator $\langle \cdot \rangle$ is referred to as the *observation operator*. The extension of this operator to tensors and tensor densities of arbitrary order is immediate from the multilinear structure of such quantities:

$$(2.13) \quad \langle T_A^B \cdots (\mathbf{x}) \rangle = \int \mathcal{H}_A^C \cdots \mathcal{L}_D^B \cdots T_C^D \cdots (\mathbf{y}) dv(\mathbf{y}).$$

The natural duality between covariant and contravariant quantities usually rubs off on the M 's and the N 's, for in most instances it turns out that the application of the observation operator to the identity tensor, $\{\delta_A^B\}$, gives a nonzero multiple of this tensor. In this way, the algebraic structure of tensor fields is almost preserved and significant elimination of pathology of a purely mathematical nature is afforded. With (2.13) and the preceding relations, we have

$$\langle \lambda(\mathbf{x}) \delta_A^B \rangle = \int \mathcal{H}_A^C(\mathbf{y}, \mathbf{x}; \mathbf{0}) \mathcal{L}_D^B(\mathbf{y}, \mathbf{x}; \mathbf{0}) \lambda(\mathbf{x}) \delta_C^D dv(\mathbf{y}),$$

and hence

$$(2.14) \quad \langle \lambda(\mathbf{x}) \delta_A^B \rangle = \langle \lambda(\mathbf{x}) \rangle \delta_A^B$$

if and only if

$$(2.15) \quad \mathcal{H}_A^C(\mathbf{y}, \mathbf{x}; \mathbf{0}) \mathcal{L}_C^B(\mathbf{y}, \mathbf{x}; \mathbf{0}) = S(\mathbf{y}, \mathbf{x}; \mathbf{0}) \delta_A^B,$$

$$(2.16) \quad \langle \lambda(\mathbf{x}) \rangle = \int S(\mathbf{y}, \mathbf{x}; \mathbf{0}) \lambda(\mathbf{y}) dv(\mathbf{y}).$$

A similar translation of (2.15) in terms of the M 's and the N 's gives

$$(2.17) \quad M_a^b(\mathbf{y}, \mathbf{x}; \mathbf{0}) N_b^c(\mathbf{y}, \mathbf{x}; \mathbf{0}) = S(\mathbf{y}, \mathbf{x}; \mathbf{0}) \delta_a^c,$$

while (2.16) defines the effect of the observation operator on scalar functions. We shall assume the validity of the relations (2.14) through (2.17), which may be viewed as a "normalization that preserves the identity". Completeness

and regularity then demand that

$$(2.18) \quad \int s(\mathbf{y}, \mathbf{x}; \mathbf{0}) \, dv(\mathbf{y}) > 0$$

for all $\mathbf{x}$ in $\mathcal{E}$.

We have consistently retained the implicit dependence of these results on the observer by the inclusion of the argument $\mathbf{0}$. This is no quirk of nicety or happenstance—rather, it entails a fundamental belief that the observer can not be removed from the picture until there is specific data that unequivocally shows such a step to be warranted.

3. From the local to the macroscopic. We denote the *local* momentum-energy tensor by $\{t_{AB}\}$ and the corresponding local metric tensor by $\{g_{AB}\}$. By a local tensor field we mean the tensor field an observer would obtain from the world structure in the immediate neighborhood of his star system. Consequently, in the first approximation obtained by ignoring the contributions to the momentum-energy tensor that obtain from the planets, comets, solar wind, solar radiation, etc., we would take the exterior Schwarzschild solution for the local metric tensor. Since the Einstein field equations are known to hold for our immediate stellar neighborhood, the g 's are solutions of the equivalent systems of field equations

$$(3.1)a \quad R_{AB}(g) - \tfrac{1}{2}R(g)g_{AB} = \eta t_{AB} ,$$

$$(3.1)b \quad R_A^B(g) - \tfrac{1}{2}R(g)\delta_A^B = \eta t_A^B ,$$

$$(3.1)c \quad R^{AB}(g) - \tfrac{1}{2}R(g)g^{AB} = \eta t^{AB} ,$$

where $\{R_{AB}(g)\}$ and $R(g)$ are the Ricci tensor and the scalar curvature formed from the metric tensor $\{g_{AB}\}$ of $\mathcal{E}$.

We can neither observe nor work directly with the local tensor fields in both galactic research and cosmological research. What is done, in effect, is to replace the local fields by certain averaged or ensemble fields in much the same fashion that continuum mechanics is obtained from point mechanics—although the analogy is closer in the case of statistical mechanics and point mechanics. It is a fact, however, that for relativistic formulations, the process of averaging or of constructing ensembles remains to be given a precise and unequivocal formulation [7].

We take as a basic tenet the validity of the Einstein field equations (3.1) for the description of a stellar neighborhood. If we apply the observation operator to the three equivalent systems of field equations (3.1), we obtain three systems of “aggregate field equations” that are not equivalent. This inequivalence arises because the observation operator does not commute, in general, with the raising or lowering of tensor indices by use of the local metric tensor and its inverse. In view of the assumptions stated by (2.14) through (2.17), the evident

choice is to base the analysis on the equations that result from application of the observation operator to the system (3.1)b. This choice effectively places the burden equally on the covariant and on the contravariant structure, and the duality of these two structures leads to a formulation that exhibits a minimal deviation from the locally valid field equations. The reader is referred to the discussion of Shirokov and Fisher given in [8] for a detailed physical argument that leads to the choice (3.1)b. Care must be exercised in applying the Shirokov and Fisher results, for they use an averaging operator that does not preserve the tensor character of the descriptors—the integral of a tensor field is not a tensor field in general. We thus obtain, with use of (2.14),

$$(3.2) \quad \langle R_A^B(g) \rangle - \frac{1}{2} \langle R(g) \rangle \delta_A^B = \eta \langle t_A^B \rangle$$

as the resulting aggregate field equations.

Now, $\{\langle t_A^B \rangle\}$ is the averaged or macroscopic momentum-energy tensor, and hence the inferable physical properties and symmetries of the aggregate would be obtained from this tensor. In fact, it is this tensor field that is assigned by physical intuition or by detailed considerations of ensembles such as those given by Synge [9]. On the other hand, if we set

$$(3.3) \quad h_{AB}(\mathbf{x}) = \langle g_{AB}(\mathbf{x}) \rangle,$$

so that $\{h_{AB}\}$ is the macroscopic or aggregate metric tensor, it is easily seen that $\{\langle R_A^B(g) \rangle\}$ is generally not the Ricci tensor formed from either $\{g_{AB}\}$ or from $\{h_{AB}\}$. A little reflection on the construction and properties of the observation operator, and the fact that the Einstein field equations are nonlinear, show that the system of field equations (3.2) that govern the aggregate have different properties than those exhibited by the Einstein field equations themselves. In particular, it is no longer true that the covariant divergence of $\{\langle t_A^B \rangle\}$, formed from the connection based on either $\{g_{AB}\}$ or on $\{h_{AB}\}$, vanishes. This follows since the contracted Bianchi identities and the Bianchi identities themselves do not hold for $\{\langle R_A^B(g) \rangle\}$. It thus follows that *the aggregate equations (3.2) do not by themselves lead to equations of motion for the aggregate.*

In general, there are three possible interpretations of this situation. The first, which we do not pursue here, is that the assumption of metrizable cannot be maintained in the description of aggregate phenomena—the affine connection of the space may have to be assumed of a more general nature than that of a metric space, such as in unified field theory [4] or in variant field theory [10, 11]. The second is that an entirely different approach must be taken to obtain the field equations for aggregates, such as that used in wave geometry [12, 13]. The third is that the metric assumption and the approach adopted here is correct, but that $\{\langle t_A^B \rangle\}$ does not adequately describe the physical interplay of the aggregate. The latter interpretation would seem to be the more likely situation, since it is well known that information is always lost in the process of averaging while the success of the Einstein field equations in cosmology indicates that the Einstein field equations cannot be too far wrong. In this

respect, if we define the *commutator tensor*, $\{C_A^B\}$, by the relations

$$(3.4) \quad C_A^B = R_A^B(h) - \langle R_A^B(g) \rangle + \frac{1}{2}[\langle R(g) \rangle - R(h)] \delta_A^B,$$

the h 's satisfy what may be referred to as the *macroscopic Einstein field equations*

$$(3.5) \quad R_A^B(h) - \frac{1}{2}R(h) \delta_A^B = \eta T_A^B$$

provided the *macroscopic momentum-energy tensor*, $\{T_A^B\}$, is defined by the relations

$$(3.6) \quad T_A^B = \langle t_A^B \rangle + \eta^{-1} C_A^B.$$

The resulting field equations (3.5) are then identical in formal structure with the Einstein field equations and the field equations of cosmology, with the exception that the metric tensor is now played by the tensor $\{h_{AB}\}$. In view of (3.3), we consequently refer to $\{h_{AB}\}$ as the *macroscopic metric tensor*.

Due to the occurrence of the tensor $\{C_A^B\}$, which will not, in general, be a null tensor, the macroscopic momentum-energy tensor $\{T_A^B\}$ will not exhibit the same physical symmetries or properties as evidenced by the observable aggregate momentum-energy tensor $\{\langle t_A^B \rangle\}$. What this says is that the constituent granular structure of an aggregate is reflected in its macroscopic representation by the presence of deviations from the geometric and physical properties expected from the observable averages of the local momentum-energy complexes. From this it is natural to interpret the quantities $\{\eta^{-1} C_A^B\}$ as the increments of macroscopic momentum-energy that are required in order to equilibrate the fluctuations present in the local geometric and momentum-energy complexes. Further, from (3.5) and (3.6) and the Bianchi identities on the metric space $\mathcal{E}^*$ with the metric tensor $\{h_{AB}\}$, one immediately obtains the *equations of motion* for the aggregate:

$$(3.7) \quad \nabla_B \langle t_A^B \rangle + \eta^{-1} \nabla_B C_A^B = 0.$$

The covariant differentiation occurring in (3.7) is formed from the Christoffel symbols of the second kind based on the metric tensor $\{h_{AB}\}$ of the space $\mathcal{E}^*$, and this space, the macrospace, is in general distinct from the metric space $\mathcal{E}$. Equation (3.7) shows that we must include the presence of the *effective force vector* $\eta^{-1} \{\nabla_B C_A^B\}$, which arises as a consequence of the granular nature of the matter that makes up the observed aggregate. This force vector accounts for the information that is lost because we are only able to observe the aggregate properties while the actual motions of the constituent elements of the aggregate are determined by the individual interactions of the constituents. From the structure-theoretic point of view, the commutator tensor and the resultant effective force vector describe the "*gestalt*" properties of aggregates whereby the properties of the whole are greater than the sum of the properties of the individual constituents.

4. Implication and inference. It was noted in the previous section that it is the tensor $\{\langle t_A^B \rangle\}$, not $\{T_A^B\}$, that is assigned for aggregates by observation

or by calculation. Thus, if we accept the working hypothesis that the Einstein field equations hold in our stellar neighborhood, the usual arguments used in cosmology [14] or in galactic research lead to the determination

$$(4.1) \quad \langle t_A^B \rangle = (\rho + p)W_A W^B - p \delta_A^B, \quad W_A = h_{AB}W^B, \quad W^A W_A = 1,$$

where ρ , p , and $\{W^A\}$ are the mean density, Poincaré pressure, and mean velocity field of the aggregate, respectively. A combination of (3.6) and (4.1) thus gives the following determination of the macroscopic momentum-energy tensor:

$$(4.2) \quad T_A^B = (\rho + p)W_A W^B - p \delta_A^B + \eta^{-1}C_A^B.$$

The construction of the macroscopic environment from the local structure thus leads to an augmentation of the "perfect fluid" momentum-energy tensor which is in no way trivial. It can not be stressed too strongly that the physical arguments used in the past to determine the whole of the momentum-energy tensor for the classical cosmological models can, in actuality, only determine $\{T_A^B - \eta^{-1}C_A^B\}$ if the Einstein field equations are to hold locally. On the other hand, if the results of the previous section are examined in detail, it can be seen that if $C_A^B \neq 0$ for some pair of the indices, positing the equality $T_A^B = \langle t_A^B \rangle$ would imply that the Einstein field equations do not hold locally. We accordingly have the following inference: *if $C_A^B \neq 0$ and the Einstein field equations hold locally, the physical conclusions obtained from the classical cosmological models based on the equality $T_A^B = \langle t_A^B \rangle$ must be reexamined in terms of the correct relations $T_A^B = \langle t_A^B \rangle + \eta^{-1}C_A^B$.* In view of the success of classical cosmology, however, it is evident that $T_A^B \cong \langle t_A^B \rangle$ is sufficient for a description of the first order effects of the gross properties of the universe as a whole.

An examination of the macroscopic field equations from the standpoint of determinism also leads to some surprises. When (3.5) and (3.6) are combined, we obtain

$$(4.3) \quad R_A^B(h) - \frac{1}{2}R(h) \delta_A^B = \eta \langle t_A^B \rangle + C_A^B.$$

We therefore have a system of ten partial differential equations of the second order that are connected by four differential identities (the contracted Bianchi identities). In the general case, the number of unknowns is thirty—the ten independent h 's, the ten independent $\langle t \rangle$'s, and the ten independent C 's—in contrast to the situation that obtains in classical cosmology wherein the number of unknowns is twenty—the ten independent h 's and the ten independent $\langle t \rangle$'s. If $\{\langle t_A^B \rangle\}$ is assumed to be given by (4.1), as is usually the case, the number of independent quantities involved in the $\langle t \rangle$'s is five—the three independent W 's, ρ , and p . In this case we would have a total of twenty five dependent variables that are connected by ten equations that must satisfy four differential identities. *If we know the local tensors $\{g_{AB}\}$ and $\{t_A^B\}$ throughout $\mathcal{E}$, we could calculate the h 's and the C 's so that the macroscopic equations would have the same degree of determinism as the local equations. The g 's and the t 's are not known throughout $\mathcal{E}$, however, since only the h 's and the $\langle t \rangle$'s are amenable to*

observation at distant points. Accordingly, the C 's must be taken as unknowns for problems involving aggregates and in classical cosmology. Thus, if the Einstein field equations hold locally, the number of unknowns in a general relativistic description of aggregate structures is ten greater than entertained in previous theories. In this context, classical cosmology may be viewed as obtained under the ten constraint equations $C_A^B = 0$, and these constraint equations may be attributed as the statement of the cosmological principle within the context of the Einstein theory.

Although the C 's are in general unknown in aggregate problems, they must satisfy the four constraint equations

$$(4.4) \quad \nabla_B \langle t_A^B \rangle = -\eta^{-1} \nabla_B C_A^B.$$

Since $\{\langle t_A^B \rangle\}$ is observable and $\{h_{AB}\}$ is the macroscopic metric tensor, observation of the motion of the aggregate would lead to a determination of the vector with the components

$$\nabla_B \langle t_A^B \rangle \stackrel{\text{def}}{=} Q_A.$$

Accordingly, the C 's must satisfy the system of linear equations

$$(4.5) \quad \nabla_B C_A^B = -\eta Q_A.$$

The general solution of (4.5) is comprised of a particular solution of (4.5) and a general solution of the homogeneous equations

$$(4.6) \quad \nabla_B Y_A^B = 0.$$

A combination of this result with (4.4) shows that $\{\langle t_A^B \rangle\}$ together with the observable motions of the aggregate can not determine the macroscopic metric structure and the macroscopic momentum-energy structure only to within solutions of the homogeneous equations (4.6). It thus follows that the indeterminism in aggregate structures and in cosmology is real and can not just be wished away.

The macroscopic metric tensor $\{h_{AB}\}$ was introduced in the previous section by the relations $h_{AB} = \langle g_{AB} \rangle$, while the averaging operator $\langle \cdot \rangle$ is basically defined in terms of its action on the local observables $\{t_A^B\}$. It is easily seen, however, that any symmetric, nonsingular tensor field could have been introduced in place of $\{h_{AB}\}$ since the local g 's are unobservable and uninferable at distant points, and the effects of such an introduction could be compensated for by the appropriate choice of the C 's. Further still, we may simply assign the h 's as per the Weyl postulates as is done in classical cosmology. In this event, equations (4.3) then serve to determine the C 's provided the $\langle t \rangle$'s are assumed known. Ultimately, what this boils down to is the fact that the h 's are artificial quantities that define a metric structure on an abstract, hyperbolic-normal, four-dimensional metric space $\mathcal{E}^*$ that need have no relation to the space-time $\mathcal{E}$ of physical events in the local Einstein theory. As an immediate consequence of these observations, we obtain the conclusions listed below.

1) The h 's may not necessarily be identified with the gravitational potentials as in the local Einstein theory.

2) The line integral of $(h_{AB} dx^A dx^B)^{1/2}$ along a geodesic in $\mathcal{E}^*$ that connects points P and Q does not determine the physical space-time interval between P and Q .

3) Since light travels along null geodesics defined by the local metric tensor $\{g_{AB}\}$, it will not travel along null geodesics defined by $\{h_{AB}\}$ unless $\mathcal{E}^*$ and $\mathcal{E}$ are conformally related; a situation that is highly unlikely.

The last conclusion requires a reinvestigation of the meaning and significance of most cosmological observables previously interpreted in terms of classical cosmology, since light does not necessarily traverse null geodesics of the classical cosmological models. It is also evident from the occurrence of objects with red shifts between one and 2.012, of a large population of quiet radio objects with ultraviolet excesses, and of rapidly pulsating radio sources, that something is awry in classical cosmological theory.

As a last remark, we note a possible alternative interpretation of the constant creation result of steady-state cosmology. As a consequence of the assumptions of the steady-state theory, it was found to be necessary to take the momentum-energy tensor as that given by (3.6) where $\{C_A^B\}$ did not vanish identically. This was because the equations of motion, (3.7), could not be reconciled with observation without $C_A^B \neq 0$. If we take the formulation of cosmological problems as presented here, we see that the tensor $\{C_A^B\}$ arises as a consequence of the averaging process whereby the homogeneity of the model is obtained from a universe comprised of galaxies and clusters of galaxies. It may therefore be argued that the constant creation prediction obtained from the model has no actual reality in that no local observer would ever observe a creation process. By this we mean that the local Einstein equations contain no creation terms; the creation process is introduced in the averaged models to reconcile the hypothetical geometry of the steady-state model with the observed properties of the universe, and simply accounts for the information lost in the averaging process. The constant-creation phenonema may be viewed solely as a consequence of introducing an averaging process whereby the real world is made to yield the line element of the steady-state model.

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