Velocity of Dislocations in Crystals

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1. Introduction. Consider an expanding dislocation loop in one of the slip planes P of a crystal and the unit vector λ giving the direction of slip in the plane P as shown in Fig. 1. The combination of a slip plane P and one of its possible slip directions λ is frequently referred to as a *slip* or *glide system*; this terminology will be used in the following discussion. At point A on the dislocation loop the outward unit normal vector ν to the loop has a direction opposite to the direction of the slip vector λ and at point C the direction of the normal ν is the same as that of the vector λ . An *edge dislocation* will therefore occur at points A and C of the loop. It is also seen that a *screw dislocation* will occur at points B and D since the vector ν is perpendicular to the slip vector λ at these points. The nature of the dislocation at other points of the loop will be determined in Section 4 from the compatibility conditions for dislocation loops.

The following article contains a theory of the velocities of the above edge and screw dislocations based on the set of nonlinear stress-strain relations which I have recently constructed for crystals containing plastic deformation, *i.e.*, the relations

$$(1) \quad \sigma_{ij} = \xi(S/S_0)e_{kk}\delta_{ij} + 2\zeta(S/S_0)e_{ij}$$

in which the σ_{ij} are the components of the stress tensor and the e_{ij} are the components of the usual strain tensor; thus

$$(2) \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

where the u_i are the components of the deformation and the comma denotes partial differentiation with respect to the coordinates of the rectangular system to which the equations (1) are referred. The coefficients ξ and ζ are scalar functions of the greatest value of the ratio S/S_0 where S is the resolved shearing stress for a glide system and S_0 is the critical value of S at points of a crystallographic region R in which the condition $S/S_0 \geq 1$ is satisfied. It will be assumed that the critical value S_0 is a constant which may be different for different glide systems and also that slip will occur over a glide system, *i.e.*, over a slip plane P and in the slip direction λ in the plane P , for which the ratio S/S_0 has its greatest value; this does not exclude the possibility that slip may also occur over other glide systems in the crystal. In particular slip may occur over a slip plane P for which the resolved shearing stress has its greatest value if S_0 is the same

FIG. 1

for all glide systems as is usually assumed. A more complete discussion of the stress-strain relations (1) will be found in my previous article [1].

The formulas which we shall derive will satisfactorily account for the experimental results obtained by Johnston, Gilman, Stein and Low on the velocities of edge and screw dislocations in the low velocity range. The problem created by dislocations having large velocities is discussed briefly at the end of this article.*

2. Compatibility conditions for dislocation loops. It is clear from the nature of the dislocation loop that there must be a discontinuity in the components of the stress and strain tensors at points of the moving loop and this discontinuity will obviously produce a similar discontinuity over a moving surface Σ , hereafter referred to as a wave surface, which has the discontinuity loop as its curve of intersection with the slip plane P . Let us now assume, as indicated in Fig. 1, that the dislocation loop is a regular curve, *i.e.*, that the loop can be represented by a set of continuously differentiable parametric functions the derivatives of which are not all zero for any value of the parameter, and similarly that the wave surface Σ is a regular surface. The latter condition implies that Σ will possess a well defined tangent plane at each of its points and hence it follows from symmetry that the surface Σ must be normal to the slip plane P along the dislocation loop.

Making the usual assumption that the components u_i of the deformation are continuous across the wave surface Σ , we have

$$(3) \quad [u_{i,j}] = \omega_i \nu_j ; \quad \left[\frac{\partial u_i}{\partial t} \right] = -G \omega_i .$$

These are the ordinary compatibility conditions for such wave surfaces. The quantities $[u_{i,j}]$ are the differences in the coordinate derivatives at contiguous points on the two sides of the surface Σ , *i.e.*, the discontinuity in the derivatives $u_{i,j}$ over the wave surface, and correspondingly $[\partial u_i / \partial t]$ denotes the discontinuity in the partial time derivatives of the deformation. The ω_i are quantities defined on the surface Σ and the ν_i are the components of the unit vector ν which is (1) normal to the surface Σ and (2) directed into the region into which the

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surface Σ is carried by its motion; finally $G(>0)$ is the normal velocity of the surface Σ in the direction of the above unit normal vector. In addition to the equations (3) we have the general dynamical conditions

$$(4) \quad \rho(G - v_n) = \bar{\rho}(G - \bar{v}_n),$$

$$(5) \quad [\sigma_{ij}]v^j = -\rho(G - v_n)[v_i],$$

in which ρ and v_n are the density and normal velocity on one side of Σ while $\bar{\rho}$ and $\bar{v}_n$ denote the corresponding quantities on the other side of this surface; the quantities $[\sigma_{ij}]$ and $[v_i]$ are, of course, the discontinuities in the stress and velocity components, respectively. Approximating v_i by the time derivative $\partial u_i / \partial t$ as is customary, it now follows from the second set of conditions (3) that

$$(6) \quad [v_i] = -G\omega_i.$$

We shall be concerned particularly with the compatibility conditions for dislocation loops, *i.e.*, compatibility conditions at the points of intersection of the wave surface Σ and the slip plane P . Now at points of the slip plane P the velocity vector will be in the plane P and will have the direction of the vector λ , or the direction opposite to the direction of the vector λ , depending on the side of the slip surface which is under consideration. At points of the dislocation loop the quantities $[v_i]$ will therefore be the components of a vector in the slip plane P and a similar remark can be made regarding the quantities ω_i on account of the equations (6). Also the vector ν normal to the wave surface Σ lies in the slip plane P at points of the dislocation loop since the surface Σ is perpendicular to the plane P along the dislocation loop as we have already observed. It follows therefore that

$$(7) \quad \omega_i n^i = 0; \nu_i n^i = 0$$

along the dislocation loop where the n^i are the components of the unit vector n normal to the slip plane P .

If we multiply the two members of the equations (1) by $n^i \lambda^j$ and sum on the repeated indices i and j , we obtain the following equation for the resolved shearing stress $S(>0)$, namely

$$(8) \quad S = \sigma_{ij} n^i \lambda^j = 2\zeta(S/S_0) e_{ij} n^i \lambda^j$$

provided the direction of the normal vector n to the slip plane is chosen appropriately and account is taken of the fact that the vectors n and λ are perpendicular. Dividing both members of (8) by S_0 and making the substitution (2), the resulting equation can be written

$$(9) \quad \frac{S/S_0}{\zeta(S/S_0)} = \frac{u_{i,j} n^i \lambda^j + u_{j,i} n^i \lambda^j}{S_0}.$$

But from the equation (9) we obtain

$$(10) \quad \left[\frac{S/S_0}{\zeta(S/S_0)} \right] = \frac{(\omega_i n^i)(\nu_j \lambda^j) + (\nu_i n^i)(\omega_j \lambda^j)}{S_0} = 0$$

when use is made of the compatibility conditions (3) and the two equations (7). Now the function $\xi(S/S_0)$ and also the function $\zeta(S/S_0)$ which occurs in the above equations (9) and (10) are material functions which must be determined, if possible, to provide an acceptable correlation between theoretical and experimental results. With this in mind the following assumption will be made. *In the immediate neighborhood of a dislocation the functions ξ and ζ are given by the equations*

$$(11) \quad \xi = \alpha(S/S_0)^\gamma; \quad \zeta = \beta(S/S_0)^\gamma$$

where $\alpha > 0$, $\beta > 0$ and $\gamma > 1$ are constants or moduli of the crystal. When the functions ξ and ζ are given by (11) the stress-strain relations (1) are intended to apply only in the immediate neighborhood of a dislocation as stated in the assumption. I wish to emphasize this fact since the extension of the relations in question to regions containing glide systems for which the resolved shearing stress exceeds the critical value but which are nevertheless devoid of dislocations, could lead to results not in agreement with observation.

The quantity appearing in the left member of the equation (9) is a monotonically decreasing function of the ratio S/S_0 in the interval

$$(12) \quad 1 \leq S/S_0 < \infty$$

as a consequence of the second equation (11). Since the interval (12) contains all possible values of the ratio S/S_0 it is clear from the condition (10) that this ratio must be continuous across the dislocation loop; this implies that the resolved shearing stress S must also be continuous across the loop since the constant S_0 depends only on the glide system under consideration. Hence the functions ξ and ζ must be continuous across the dislocation loop, *i.e.*,

$$(13) \quad [\xi(S/S_0)] = 0; \quad [\zeta(S/S_0)] = 0.$$

It follows from the stress-strain relations (1) and the conditions (13) that

$$(14) \quad [\sigma_{ij}] = \xi[e_{kk}]\sigma_{ij} + 2\zeta[e_{ij}].$$

Now multiply the equations (14) through by ν^j , sum on the repeated index j , and then eliminate the left members of the resulting equations by the substitution (5); when use is also made of the compatibility conditions (3) and the equations (6) we are led to the following set of equations

$$(15) \quad \rho G(G - v_n)\omega_i = (\xi + \zeta)\omega_k \nu^k \nu_i + \zeta \omega_i.$$

But the ω_i are proportional to the components λ_i of the slip direction λ as we have seen and hence the equations (15) can be written

$$(16) \quad \rho G(G - v_n)\lambda_i = (\xi + \zeta)\lambda_k \nu^k \nu_i + \zeta \lambda_i.$$

The equations (16) which are valid along the dislocation loop will be used in the next section to obtain the desired formulas for the velocities of the edge and screw dislocations.

3. Velocity of dislocations. The scalar product $\lambda_k \nu^k$ does not vanish at points *A* and *C* of the dislocation loop (see Fig. 1) since the vector components λ_k and ν^k are proportional at these points. Hence if we multiply the two members of the equations (16) by ν^i and sum on the repeated index *i*, we find that

$$(17) \quad \rho G(G - v_n) = \xi + 2\zeta.$$

The equation (17) gives the velocity *G* of the edge dislocation. To obtain the velocity *G* of the screw dislocation we make use of the fact that the scalar product $\lambda_k \nu^k$ vanishes at the points *B* and *D* in Fig. 1. Hence the equations (16) reduce to

$$(18) \quad \rho G(G - v_n) = \zeta$$

at points *B* and *D*; the equation (18) accordingly gives the velocity *G* of the screw dislocation. Finally if we neglect the normal material velocity v_n in comparison with the wave velocity *G* and replace ξ and ζ by their values as given by (11), the equations (17) and (18) become

$$(19) \quad \rho G^2 = (\alpha + 2\beta)(S/S_0)^\gamma; \quad \rho G^2 = \beta(S/S_0)^\gamma$$

and the following result can be stated. *The velocities G of the edge and screw dislocations are given, respectively, by the first and second equations (19).* It will be shown in Section 5 that the equations (19) will account for the observed behavior of the edge and screw dislocation velocities in the low velocity range as mentioned in the introduction.

4. Structure of dislocation loops. Neglecting the normal velocity v_n in comparison with the wave velocity *G* as in Section 3, the equations (16), (17) and (18) become

$$(20) \quad (\rho G^2 - \zeta)\lambda_i = (\xi + \zeta)(\lambda_k \nu^k)\nu_i,$$

$$(21) \quad \rho G^2 = \xi + 2\zeta,$$

$$(22) \quad \rho G^2 = \zeta,$$

respectively. Now the procedure used in Section 3 shows that the equation (21) is valid in any segment *M* of the dislocation loop in which the scalar product $\lambda_k \nu^k$ is different from zero. Eliminating the quantity ρG^2 from (20) by means of (21) we have

$$(23) \quad (\xi + \zeta)\lambda_i = (\xi + \zeta)(\lambda_k \nu^k)\nu_i.$$

But the quantity $\xi + \zeta$ does not vanish as we see from the equations (11) and hence the equations (23) reduce to

$$(24) \quad \lambda_i = (\lambda_k \nu^k)\nu_i.$$

It follows from (24) that the vectors λ and ν have the same (or opposite) directions. *In other words the above segment M is perpendicular to the slip direction λ .* If the scalar product $\lambda_k \nu^k$ is equal to zero in a segment *N* of the dislocation loop,

the following result can be stated immediately. *The segment N is parallel to the slip direction λ .*

It is clear from the above italicized statements that the dislocation loop must be a broken line constructed from alternating M and N segments as indicated in Fig. 2. This result has been surmised by various authors from other considerations and the appropriate references to the literature can be found in the book by Cottrell [2] where the structure of the dislocation loop has been discussed.

In Section 2 the regularity of the dislocation loop and the associated wave surface Σ was assumed and this assumption appeared to be entirely proper. It is now seen that the loop and wave surface can at best be piece-wise regular; but this result in no way affects the analytical procedures or the validity of the formulas for the velocities of the edge and screw dislocations which were derived under the assumption of complete regularity.

5. Correlation between theory and experiment. In Fig. 3 we have shown the graphical relationship between the edge and screw velocities G of a dislocation loop and the resolved shearing stress S ; the graphs in this figure were obtained from measurements by Johnston and Gilman [3] on a lithium fluoride crystal. Logarithmic scales to the base 10 are used to express their results and hence the curves in Fig. 3 can be regarded as the graphs which directly relate the logarithms of the velocities G and the logarithm of the stress S when appropriate conventional scales are introduced. By inspection of these graphs it is seen that they are parallel in the lower velocity range.

The effect of temperature changes on the edge dislocation velocities of specimens of silicon-iron crystals was measured by Stein and Low [4] and the results are shown by the graphs in Fig. 4. It is seen that the graphs are linear and parallel when log scales are used, as in Fig. 3, to represent the relationship between the above velocities and the resolved shearing stress. Analogous results appear to occur for screw dislocation velocities under temperature changes and also for the velocities of edge and screw dislocations after a crystal has been subjected to radiation damage. This is illustrated by the graphs in Figs. 5 and 6 which

FIG. 2

FIG. 3

appear in an article by Gilman [5] and which were constructed from measurements on lithium fluoride crystals. The fact that the graphs in Fig. 5 are not exactly parallel may possibly be attributed to experimental error; in any case if one restricts the velocity range by excluding the two uppermost observational

FIG. 4

points on the graph for the velocities at 300°K, the resulting graph will be parallel to the other graph in the figure. The graphs in Fig. 6 are closely parallel. *It may be concluded from these observations that, within a restricted velocity range, the graphs giving the edge and screw dislocation velocities are parallel and that these graphs remain parallel under changes of temperature and the effects of radiation damage.* In the case of lithium fluoride crystals a study of the graphs appears

to indicate that this result is valid for screw dislocation velocities not exceeding 10^{-2} cm/sec and for edge dislocation velocities not greater than $\frac{1}{2}$ cm/sec; the larger velocity range for the edge velocities is due to the fact that these velocities are approximately 50 times greater than the corresponding velocities of the screw dislocations as stated by Johnston and Gilman [3].

The two equations (19) can be written respectively, as

$$(25) \quad \log G = \frac{\gamma}{2} \log S + \frac{1}{2} \log \left(\frac{\alpha + 2\beta}{\rho} \right) - \frac{\gamma}{2} \log S_0 ,$$

$$(26) \quad \log G = \frac{\gamma}{2} \log S + \frac{1}{2} \log \frac{\beta}{\rho} - \frac{\gamma}{2} \log S_0 .$$

But $\gamma/2$ gives the slope of any one of the linear velocity-stress graphs and hence it follows from the last italicized statement that the modulus γ is independent of temperature changes and radiation damage. Now suppose that the modulus γ has been determined for the crystal under consideration. Let us also suppose that the density ρ may be regarded as constant and independent of the tempera-

FIG. 5

FIG. 6

ture and radiation effects. The critical value S_0 is to be found experimentally after the crystal has been heated or cooled to the desired temperature and damaged by radiation. Then the linear graphs giving the velocities of the screw dislocations can be represented accurately by the equation (26) for an appropriate choice of the modulus β ; we denote this modulus by $\beta(T, R)$ to indicate that it may possibly depend on temperature and radiation damage. Similarly the equation (25) will give an accurate representation of the graphs for the edge dislocation velocities when a suitable choice of the modulus $\alpha(T, R)$ has been made. In reaching the latter conclusion it is supposed that the modulus $\beta(R, T)$ has previously been determined by the foregoing procedure. Returning to the form (19) of the velocity-stress equations let us now write these equations as

$$(27) \quad \rho G^2 = [\alpha(T, R) + 2\beta(T, R)](S/S_0)^\gamma,$$

$$(28) \quad \rho G^2 = \beta(T, R)(S/S_0)^\gamma,$$

respectively, to indicate the possible dependence of the moduli α and β on temperature and radiation.

The critical value S_0 of the resolved shearing stress S can be regarded, roughly speaking, as the least value of S which will produce an observable motion of the entire dislocation loop. To be more precise let us define S_0 as that value of the resolved shearing stress which will impart a barely discernable velocity G_0 to a screw dislocation. While the appropriate selection of the velocity G_0 had best be left to the experimenter it is obvious that G_0 must be extremely small, *e.g.* 10^{-6} or 10^{-7} cm/sec. As so defined the stress S_0 will be just sufficient to cause the motion of the entire dislocation loop; this follows from the fact that the edge dislocation components of the loop move with greater velocity than the screw dislocation components.

Now consider the various actual or hypothetical graphs given by the heavy lines in Fig. 7; it is to be understood that log scales are used in the construction of these graphs although the actual scales have not been indicated explicitly. The graphs in this figure are assumed to represent the relationship between the screw dislocation velocity and the stress in specimens of a crystal at different temperatures and different amounts of radiation damage. Each graph will end along the line L_0 determined by the above velocity G_0 as shown in the figure and the stresses corresponding to these end points will be the critical stresses S_0 associated with the various graphs. Let us also consider the line L_1 in Fig. 7 which intersects the graphs at points where the screw dislocation velocity has the common value G_1 . Denoting by S the resolved shearing stresses at these points of intersection, the relationship between S and the critical value S_0 for any graph will be given by the equation (28), *i.e.*, by

$$(29) \quad \beta(T, R)(S/S_0)^\gamma = \rho G_1^2 = \text{const.}$$

in which the value of the modulus $\beta(T, R)$ is determined by the graph under consideration. But since the graphs in Fig. 7 are parallel it is seen immediately that

$$\log S - \log S_0 = \log (S/S_0) = \text{const.}$$

FIG. 7

where the right member is an absolute constant, *i.e.*, it is the same for all graphs. Hence the ratio S/S_0 is an absolute constant and it follows therefore from (29) that the modulus $\beta(T, R)$ is also such a constant or, in other words, that $\beta(T, R)$ is independent of the temperature and the radiation damage to which the crystal has been subjected.

Corresponding to each of the graphs in Fig. 7 there will be a linear graph giving the relationship between the edge dislocation velocity and the resolved shearing stress. We have drawn these graphs in Fig. 8 and also the graph marked *AB* which is taken from Fig. 7; as in Fig. 7 the graphs in Fig. 8 are referred to log scales without explicit indication. Use the graph *AB* to determine the line L'_0 as shown in the figure. Also draw the horizontal line L'_1 so as to intersect the various graphs for the edge dislocation velocities. When we now replace the equation (28) by the equation (27) and the lines L_0 and L_1 in the preceding discussion by the lines L'_0 and L'_1 in Fig. 8, we are led to the equation

$$\alpha(T, R) + 2\beta(T, R) = \text{const.}$$

in which the right member is an absolute constant. Since $\beta(T, R)$ has already been shown to be an absolute constant it follows from the above equation that $\alpha(T, R)$ must likewise be an absolute constant. The following result has now been established. *The moduli α , β and γ are constants which are independent of temperature changes and radiation damage to the crystal.* Temperature changes and radiation damage can therefore affect only the critical value of the resolved shearing stress.

It suffices from the above result to determine the moduli α , β and γ in the velocity-stress equations (19) by measurements at room temperature on a crystal which has not been damaged by radiation. Eliminating the functions $\xi(S/S_0)$

FIG. 8

and $\zeta(S/S_0)$ from (1) by means of the equations (11) we find that the stress-strain relations in the neighborhood of a dislocation are given by

$$(30) \quad \sigma_{ij} = \alpha(S/S_0)^\gamma e_{kk} \delta_{ij} + 2\beta(S/S_0)^\gamma e_{ij}$$

in which it can now be stated that the moduli α , β and γ are independent of temperature and radiation damage.

Some difficulties may arise in the problem of formulating a theory which will account for the velocity of dislocations in the higher velocity range. It would of course be a simple matter to choose the functions $\xi(S/S_0)$ and $\zeta(S/S_0)$ so that the equations (19) will represent the velocity-stress curves in Fig. 3 to any degree of approximation. But this may not be the correct approach from the physical standpoint. It is conceivable, for example, that the higher stresses which are involved in the upper portion of the velocity-stress curves in Fig. 3 will produce such structural changes that the physical requirements of the problem can best be met by replacing the stress-strain relations (1) by relations between velocities and rates of stress. As matters stand it seems to me that there is at present insufficient observational data for the proper evaluation of this problem and I do not believe that a serious attempt should be made to find a solution until the requisite data is available. In the meantime one can use the stress-strain relations (30) in the investigation of problems involving dislocations in stress fields for which the resolved shearing stress is not too large in comparison with its critical value.

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