Corrections and Additions to “The Mechanical Foundations of Elasticity and Fluid Dynamics”

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Acknowledgement. I am grateful for the corrections supplied by a number of correspondents, especially Dr. Ericksen, Professor Kuerti, Mr. R. Toupin, and Professors Whaples and Wundheiler. Some portions which have proved slow reading are expanded a little. On this occasion the article (except for §32) is brought up to date, to the best of my knowledge, as of May, 1953. No further additions are to be published, but since the material is being reworked in a new form, I should be greatly obliged for notice of any remaining errors and omissions.

P. 127, l. 13, replace comma after “history” by semi-colon.

P. 127, l. 14, replace period after “history” by comma and add: except that there is a residual hydrostatic pressure appropriate to the density and temperature.

P. 129, footnote 1, insert: [1924, 8] [1925, 6] [1928, 8] [1948, 60] [1951, 22].

P. 131, l. 3 after (6.3), instead of a_i read a_r .

P. 131, replace ll. 1–3 above (6.4) by:

If the tensor a_{ij} is symmetric, i.e. if $a_{ij} = a_{ji}$, it does not generally follow that the matrix $\|a^i_j\|$ is symmetric; however, it does so follow in a rectangular Cartesian system, and we shall be concerned only with Euclidean three-dimensional space, where such a system is always available, so that, since the proper values a_r are scalars, we can carry over to symmetric tensors those properties of symmetric matrices which can be proved by use of the proper values only. For example, the proper values of a real symmetric tensor are real, and it is always possible to choose a set of three real orthogonal unit proper vectors n_r^i :

$$a^i_j n_r^j = a_r n_r^i, \quad g_{ij} n_r^i n_\Delta^j = \delta_{r\Delta}, \quad (6.3a)$$

where g_{ij} is the metric tensor. Since the vectors n_r^i are linearly independent, the matrix $\|n_r^i\|$ possesses a unique inverse, whose entries we shall write n^Δ_j , so

that $n_{\Gamma}^i n_{\Delta}^i = \delta_{\Gamma}^{\Delta}$. When the proper values of $\mathbf{a}$ are all positive, the *positive* r^{th} power $\mathbf{a}^r$ for any real r is that symmetric tensor whose principal axes co-incide with those of $\mathbf{a}$ and whose proper values are the positive r^{th} powers of the a_{Γ} . It is easy to verify that this definition satisfies the laws of exponents and reduces to the more familiar one when r is an integer.^{3a} A formula expressing it is

$$(a^r)^i_j = (a_{\Gamma})^r n_{\Gamma}^i n^{\Gamma}_j. \quad (6.3b)$$

Now an analytic function of the components of an $n \times n$ symmetric matrix $\mathbf{a}$ is a scalar⁴ if and only if it be a function of the n *principal invariants* of $\mathbf{a}$. These invariants may be expressed as the elementary symmetric functions of the proper values, for the case $n = 3$ assuming the form:⁵

^{3a} A more general and more complicated definition of powers of a matrix may be inferred from [1934, 1, §§8.05-8.07].

P. 131, footnote 1, replace [1939, 12] by [1930, 12].

P. 132, l. 1. Begin new ¶, and before it insert:

The sufficiency of the conditions stated is obvious. The necessity may be proved by considering the class of orthogonal transformations only. By such a transformation the off-diagonal terms may be annulled: $f(\mathbf{a}) = f(a_1, a_2, \dots, a_n)$. Since further orthogonal transformations can permute the a_{Γ} in any way, f must be a symmetric function of the a_{Γ} , and hence a function of the n elementary symmetric functions.

P. 132, in (6.10) replace i by Γ and j by Δ in the sums on the right, replace period at end by comma, and add:
and in general the m^{th} moment $\bar{I}_{(m)\mathbf{a}}$ is defined by

$$\begin{aligned} \bar{I}_{(m)\mathbf{a}} &\equiv a^{i_1}_{i_2} a^{i_2}_{i_3} \cdots a^{i_m}_{i_1} = \sum_{\Gamma} (a_{\Gamma})^m, \\ &= (-)^{m-1} m \sum_{i=1}^m (-)^{i-1} (i-1)! \sum \frac{I_a^j II_a^k III_a^l}{j! k! l!}, \end{aligned} \quad (6.10a)$$

where the last expression is valid only for $n = 3$, the second sum being understood to run over all values of j, k, l such that $j + k + l = i, j + 2k + 3l = m$.

P. 132, footnote 3, replace [1952, 3] by [1953, 1].

P. 132, footnote 4, replace last sentence by: G. WHAPLES has kindly communicated to me a proof, too long to include here, that for non-symmetric mixed tensors a^i_j under arbitrary co-ordinate transformations the result remains valid. For the non-symmetric case, however, it is more useful in continuum mechanics to consider scalar functions of all four matrices $\|a_{ij}\|$, $\|a^i_j\|$, $\|a_j^i\|$, $\|a^{ij}\|$, or, equivalently, of the two matrices $\|g_{ij}\|$, $\|a^{ij}\|$. In a Riemannian space it is always possible to choose an infinity of orthogonal co-ordinate systems such that at an assigned point $g_{ij} = \delta_{ij}$, and therefore it is sufficient to find the most general function of a non-symmetric matrix which is a scalar with respect to orthogonal transformations. For the case $n = 3$ this problem was solved by RABINOVITCH [1913, 7], who found that beside I_a, II_a, III_a four more invariants are required, and that

these invariants are connected by one scalar relation, not in general soluble for any one as a single-valued function of the others.

P. 132, footnote 7, replace period at end by comma and add: and both the invariance and the significance of Δ_a were known to MAXWELL in 1856 [1937, 10, pp. 32-3].

P. 134, l. 2 after (7.2), replace "system" by "system²".

P. 134, bottom, add:

² (7.3) is a consequence of (6.6). Further relations between various invariants of **a** and **b** are obtained by NOVOZHILOV [1951, 27, §5].

P. 138, footnote 1, add at end: Relations between the three classes of differential invariants are discussed by WUNDHEILER [1932, 16], whose work is generalized by HLAVATÝ [1933, 13].

P. 139, (12.3), replace $\frac{\partial x^i}{\partial X_\alpha}$ by $\frac{\partial x^i}{\partial X^\alpha}$.

P. 140, the unnumbered display equation should be (13.4).

P. 141, delete the second paragraph through l. 3 of p. 142 and replace by Equivalent to (14.3)₁ and (14.3)₂ are

$$C^L{}_M = G^{LN} x^i{}_{,N} g_{ij} x^j{}_{,M} = x^{i,L} x_{i,M}, \quad c^i{}_j = X^{L,i} X_{L,j}, \quad (14.5)$$

and by (13.2) it is easy to see that³

$$(C^{-1})^L{}_M = X^{L,i} X_{M,i}, \quad (c^{-1})^i{}_j = x^{i,L} x_{j,L}. \quad (14.6)$$

The theory of finite strain is the creation of Cauchy.⁴ The set of differential elements dX^L which at a fixed particle X^P sweep out a sphere of radius k are carried by the motion (12.2) into a set of elements dx^i at the place x^k occupied by X^P at time t ; by (14.1) and (14.2), these elements dx^i sweep out the quadric surface $k^2 = G_{LM} dX^L dX^M = c_{ij} dx^i dx^j$. Conversely, the elements dx^i which at time t and place x^k sweep out a sphere of radius k were at time t_0 elements dX^L sweeping out the quadric surface $k^2 = g_{ij} dx^i dx^j = C_{LM} dX^L dX^M$. Since **c** and **C** are real and symmetric, their proper values c_r and C_r are real; proper vectors corresponding to distinct proper values are orthogonal; and it is possible to choose a real orthogonal material co-ordinate system in which $C^L{}_M$ is a diagonal matrix with entries C_r , and a real orthogonal spatial co-ordinate system in which $c^i{}_j$ is a diagonal matrix with entries c_r . Since **g** and **G**, being real Euclidean metric tensors, are positive definite, it follows that $c_r > 0$, $C_r > 0$. Hence the above quadrics are ellipsoids. They may be called the material and spatial *strain ellipsoids*; any set of principal axes of the material strain ellipsoid is called a set of *principal material axes of strain*, and *principal spatial axes of strain* are analogously defined. If the proper values are distinct, the principal axes are uniquely defined.

Using the notations of §6 for orthogonal unit proper vectors, we have $c^i{}_j n_r^j = c_r n_r^i$, or, equivalently, $c_{ij} n_r^i = c_r g_{ij} n_r^j$. Put

$$N_r^L \equiv X^{L,i} n_r^i / \sqrt{c_r}. \quad (14.6a)$$

Then by (14.3) and (13.2) follows

$$\begin{aligned}
 C_{LM} N_{\Gamma}^M &= g_{ij} x^i_{,L} x^j_{,M} X^M_{,k} n_{\Gamma}^k / \sqrt{c_{\Gamma}} = g_{ij} x^i_{,L} n_{\Gamma}^j / \sqrt{c_{\Gamma}}, \\
 &= (c_{\Gamma})^{-1} c_{ij} x^i_{,L} n_{\Gamma}^j / \sqrt{c_{\Gamma}}, \\
 &= (c_{\Gamma})^{-1} G_{MN} X^M_{,i} X^N_{,j} x^i_{,L} n_{\Gamma}^j / \sqrt{c_{\Gamma}}, \\
 &= (c_{\Gamma})^{-1} G_{LM} X^M_{,j} n_{\Gamma}^j / \sqrt{c_{\Gamma}} = (c_{\Gamma})^{-1} G_{LM} N_{\Gamma}^M.
 \end{aligned} \tag{14.6b}$$

Hence $(c_{\Gamma})^{-1}$ is a proper value of $\mathbf{c}$ and N_{Γ}^L is a corresponding proper vector. Moreover,

$$\begin{aligned}
 G_{LM} N_{\Gamma}^L N_{\Delta}^M &= G_{LM} X^L_{,i} X^M_{,j} n_{\Gamma}^i n_{\Delta}^j / \sqrt{c_{\Gamma} c_{\Delta}}, \\
 &= c_{ij} n_{\Gamma}^i n_{\Delta}^j / \sqrt{c_{\Gamma} c_{\Delta}} = c_{\Gamma} g_{ij} n_{\Gamma}^i n_{\Delta}^j / \sqrt{c_{\Gamma} c_{\Delta}}, \tag{14.6c} \\
 &= \delta_{\Gamma\Delta}.
 \end{aligned}$$

Thus it is possible to order the C_{Γ} in such a way that $c_{\Gamma} = C_{\Gamma}^{-1}$ (and hence follows⁵ $c_{\Gamma} = (C^{-1})_{\Gamma}$, $C_{\Gamma} = (c^{-1})_{\Gamma}$) and that the vectors N_{Γ}^L , defined by (14.6a), form a corresponding orthogonal triple of unit proper vectors of $\mathbf{C}$. Hence the squares of the lengths of corresponding semi-axes of the two strain ellipsoids are reciprocal to one another. Now from (14.2) it follows that if a vector of length L_{Γ} tangent to a principal axis of $\mathbf{C}$ be carried into a vector of length $L_{\Gamma} + \Delta L_{\Gamma}$ then $(1 + \Delta L_{\Gamma}/L_{\Gamma})^2 = C_{\Gamma}$. Similarly, if a vector tangent to the corresponding principal axis of $\mathbf{c}$ be now of length $l_{\Gamma} + \Delta l_{\Gamma}$, but originally of length l_{Γ} , then $(1 + \Delta l_{\Gamma}/l_{\Gamma})^{-2} = c_{\Gamma}$. Our result shows that $\Delta L_{\Gamma}/L_{\Gamma} = \Delta l_{\Gamma}/l_{\Gamma}$; this common value is the *principal extension* δ_{Γ} corresponding to the principal direction.

Thus the entire displacement in the neighborhood of a particle may be regarded as the succession of a translation, the rotation of the material strain axes into the spatial ones, and extensions of these three axes in the ratios $1 + \delta_{\Gamma} = \sqrt{C_{\Gamma}} = \sqrt{c_{\Gamma}^{-1}}$. It remains only to express this resolution analytically. Using the notation of §6, put

$$r^L_{\Gamma i} \equiv N_{\Gamma}^L n^{\Gamma}_{\Gamma i}, \quad R^i_L \equiv n_{\Gamma}^i N^{\Gamma}_L. \tag{14.6d}$$

Then $\mathbf{R} = \mathbf{r}^{-1}$, and also

$$n_{\Gamma}^i = R^i_L N_{\Gamma}^L, \quad N_{\Gamma}^L = r^L_{\Gamma i} n_{\Gamma}^i. \tag{14.6e}$$

Consequently $\mathbf{R}$ rotates the orthogonal unit triple N_{Γ}^L into the orthogonal unit triple n_{Γ}^i , while $\mathbf{r}$ represents the inverse rotation. Now by (14.6a) and (14.6e) we have

$$\begin{aligned}
 X^L_{,i} &= X^L_{,j} n_{\Gamma}^j n^{\Gamma}_{\Gamma i} = \sqrt{c_{\Gamma}} N_{\Gamma}^L n^{\Gamma}_{\Gamma i}, \\
 &= r^L_{\Gamma j} \sqrt{c_{\Gamma}} n_{\Gamma}^j n^{\Gamma}_{\Gamma i}.
 \end{aligned} \tag{14.6f}$$

By (6.3b) follows then the former of the two desired results:^{5a}

$$X^L_{,i} = r^L_j (c^\dagger)^j_{,i}, \quad x^i_{,L} = R^i_M (C^\dagger)^M_L. \quad (14.6g)$$

Thus the rotations may be calculated in terms of the displacement gradients as follows:

$$r^L_j = X^L_{,i} (c^{-\dagger})^i_j, \quad R^i_M = x^i_{,L} (C^{-\dagger})^L_M. \quad (14.6h)$$

By (6.4) and the foregoing results we have⁶

P. 141, footnote 3, delete the first and third sentences, replace the first by:

⁴ [1827, 2] [1841, 1]; for part of the proofs given here I am indebted to Mr. R. TOUPIN.

P. 141, footnote 4, number this footnote 3, and add: BONVICINI [1932, 10] [1935, 3] discusses the properties of $C_{LM,N}$.

P. 142, The first display equation should be numbered (14.7). In the line following, replace the subscript i by Γ (four times).

P. 142. Add footnote: ^{5a} Cf. [1924, 8, §7] [1925, 6, §15].

P. 143, replace last two lines before §15 by:

cluded is that $I_{c^{-1}}$, $II_{c^{-1}}$, $III_{c^{-1}}$ are positive and differ from 3, 3, and 1, respectively, by a quantity which is $O(\delta)$.

P. 144, (15.3) replace second $III_{c^{-1}}$ by III_c^{-1} .

P. 145, last line of text, replace *compatability* by *compatibility*.

P. 145, footnote 1, l. 8, to references add [1947, 30].

P. 145, footnote 1, l. 4 from end, replace "states that" by "states [1950, 21] that".

P. 146, l. 2 of §19, replace "strains." by "strains.^{1a}"

P. 146, last line of text, replace $3\frac{1}{2}$ by $\frac{3}{2}$.

P. 146, last two lines of first footnote, add references: [1948, 60, §39] [1951, 20].

P. 146, after last line add:

^{1a} Some of the material of this section is presented in greater detail by Novozhilov [1948, 60, §§14–15], who points out weaker sufficient conditions and intermediate approximations.

P. 147, l. 1, replace $3\frac{1}{2}$ by $-\frac{3}{2}$.

P. 147, l. 5 after (19.3), add: Moreover, (14.6h) may be approximated by Cauchy's^{2a} linear expressions

$$\tilde{r}_{ij} \equiv \frac{1}{2}(u_{i,j} - u_{j,i}); \quad (19.3a)$$

i.e., $r_{xx} \approx R_{xx} \approx \tilde{r}_{xx} + 1$, $r_{xy} \approx R_{xy} \approx \tilde{r}_{xy}$ etc., and the formal identity

$$u_{i,j} = \tilde{e}_{ij} + \tilde{r}_{ij} \quad (19.3b)$$

can be interpreted as an additive resolution of the displacement gradient into

pure strain and rotation,^{2a} replacing, in this approximation, the general multiplicative formulae (14.6g).

P. 147, after footnote 2 add: ^{2a} [1841, 1].

P. 149, (20.1), the first symbol should be $\dot{b}$.

P. 149, l. 1 after (20.1), read Γ_{jk}^i .

P. 150, at end of §20, insert:

Suppose a certain vector field $\mathbf{b}(x^i, t)$ be invariant under the motion (12.2). Then the quantities b^i must transform as contravariant components of a vector field when (12.2) is regarded as a transformation of co-ordinates:

$$b^i = x^i_{,\alpha} B^\alpha. \quad (20.5)$$

Then $\dot{b}^i = \dot{x}^i_{,\alpha} B^\alpha = \dot{x}^i_{,j} x^j_{,\alpha} B^\alpha = \dot{x}^i_{,j} b^j$; and conversely, if this last relation holds for all time, we shall regain (20.5) by integration. Define the *absolute time flux* $\mathbf{b}'$ of $\mathbf{b}$ by

$$b'^i \equiv \dot{b}^i - \dot{x}^i_{,j} b^j. \quad (20.6)$$

Then we have shown that a *vector field is invariant under the motion if and only if its absolute time flux vanish*. A similar result holds for tensors of all orders.

P. 152, (22.5), replace $\log \frac{\dot{\cdot}}{\cdot}$ by $\log \frac{\dot{\cdot}}{\dot{\cdot}}$.

P. 154, after §22 add:

Equivalent to (13.5)₂, for a fixed reference configuration, is $JdA_\alpha = da_i x^i_{,\alpha}$. Material differentiation of this expression yields a result which when simplified by (22.6)₂ and (13.5)₁ is equivalent to⁶

$$\frac{\dot{\cdot}}{da_i} = -\dot{x}^j_{,i} da_j + \dot{x}^j_{,j} da_i. \quad (22.16)$$

P. 154, before present footnotes add: ⁶ [1877, 2].

P. 154, l. 3 of second footnote 1, replace [1952, 4, §7] by [1954, 1].

P. 159, (26.9)₂, replace $X^L_{,j}$ by $X^L_{,i}$.

P. 159, (26.13)₁, replace $\dot{x}^i$ by $\dot{x}_i$.

P. 159, at end of footnote 12 add:

Various approximate forms of (26.8) and (26.10) occur in the literature; e. g. [1948, 60, §§21–22].

P. 160, l. 2, replace “Meyer” by “Mayer”.

P. 160, l. 2 above (27.2), between “by” and “(26.2)” insert: (24.5)₂ and.

P. 160, l. 6 of footnote 1, between “false” and “cf.” insert: as PARTINGTON [1949, 47, Part II, §33, last footnote] has remarked, the presentation of LIPPMANN [1889, 7, pp. 76–78] shows that CARNOT’s argument becomes consistent with thermodynamics if “calorique” be interpreted as entropy;

P. 160, last l. of footnote 1, add: [1943, 21].

P. 160, footnote 3, add: The assertions of WATERSTON [1843, 4] are somewhat more restricted because of his insistence on deriving his results from a molecular model.

P. 161, replace ll. 5–6 by:
specialization into (27.1), by using (24.5)₂, (22.5)₂, and (26.2)₁ we obtain in place of (27.2) the classical formula

P. 161, replace (28.2) and the line above it by:
where $\mathfrak{V}$ is the volume of the body $\mathfrak{U}$,

$$d\mathfrak{E} \equiv \dot{\mathfrak{E}}dt, \quad d\mathfrak{B} \equiv \dot{\mathfrak{B}}dt, \quad \delta\mathfrak{Q} \equiv - \left[\oint_{\mathfrak{S}} q^i da_i \right] dt. \quad (28.2)$$

P. 161, l. 3 of footnote 1, between the second and third sentences insert: A different system of thermodynamics, largely unpublished, was developed by WATERSTON from 1843 onward. WATERSTON's viewpoint, rather than that of CARNOT, KELVIN, and CLAUSIUS, is regarded as correct by HALDANE [1928, 7, Ch. II], who develops and applies it. The difference of opinion, of course, lies not in the mathematics but in the choice of physical principles.

P. 161, l. 5 of footnote 1, add: [1947, 29] [1949, 56–57] [1951, 17] [1952, 11–12, 19].

P. 161, last line of footnote 1, add: [1952, 4].

P. 162, l. 5, replace $\mathfrak{S}$ by $d\mathfrak{S}$.

P. 163, l. 3 from end of footnote 5, replace “council” by “counsel”.

P. 164, in l. 1 of footnote 1, between “obtains” and “an” insert: for pure gases.

Pp. 166–168. §32 as it stands is quite incomplete. To bring it into satisfactory condition would exceed the scope of this note, so it is preferable simply to delete it.

P. 174, footnote 9, after [1925, 1] add: [1925, 5, §§4–12].

P. 174, footnote 10, at end add: Cf. [1952, 9, 13].

P. 176, footnote 2, add at end of part A: ERICKSEN [1953, 5] shows that in an incompressible body acceleration waves are always transverse, and that if Σ satisfy (41.13a) then the speeds of propagation are real whenever the wave surface is normal to one of the principal spatial axes of strain.

P. 177, (39.3), replace $C_{\alpha\beta}$ by C^{α}_{β} .

P. 180, after footnote 3 add: The relations between various invariants of stress and strain which are implied by (41.6) are discussed by NOVOZHILOV [1951, 27, §3].

P. 181, l. 1 above (41.11), replace “energy:” by “energy:^{7a}”.

P. 181, after footnote 7, add: ^{7a} A special case was derived by HOUSTOUN [1911, 12].

P. 181, footnote 9, add: Essentially the same result, in a far less convenient form, was obtained by ARIANO [1930, 19, pp. 753–754], except that he neglected to observe the arbitrariness of the hydrostatic term.

P. 182, l. 3, replace “conditions.” by “conditions.^{9a}”.

P. 182, after footnote 9, add: ^{9a} ERICKSEN [1953, 5] shows that these conditions are necessary and sufficient in order that the speeds of propagation of acceleration waves normal to the principal spatial axes of strain be real.

P. 182, footnote 11, add: [1952, 6].

P. 182, footnote 13, insert: [1924, 6, §8] [1930, 19, pp. 742-743] [1933, 12].

P. 184, l. 2 after (42.7), replace "circumstances." by "circumstances.^{1a}"

P. 184, (42.8)₂, delete 2 in denominator.

P. 184, bottom, add: ^{1a} Cf. [1934, 5].

P. 190, l. 5, replace "*traction*," by "*traction*.^{9a}"

P. 190, bottom, add:

^{9a} The necessity for such end stresses was apparently first noticed by RIZ [1938, 12], who used very special stress-strain relations (§50). Earlier the shortening which accompanies large pure torsion of a compressible rectangular beam had been observed in the engineering literature, and a simple special theory was constructed for it [1914, 3] [1921, 3, §§20-26] [1927, 6] [1936, 7] [1949, 58] [1951, 23].

P. 190, l. 5 after (42.25), between "*Bending*" and "*a*" insert: *of*.

P. 192, footnote 16, add: [1952, 6, 16]. The experimental results given in these papers confirm in detail the predictions of the general theory of elasticity in a great variety of situations.

P. 193, replace first paragraph by:

Green & Shield¹⁷ solve the problem of small twist k of a prism of arbitrary isotropic elastic material subjected to arbitrary biaxial extension. The deformation is

$$\begin{aligned}x &= KX - kKDYZ, & y &= KY + kKDXZ, \\z &= DZ + kD^2 w(X, Y),\end{aligned}\tag{42.34}$$

where D and K are the longitudinal and lateral extensions and $w(X, Y)$ is the classical torsion function for the cross-section. Among the results is the general formula for the torsional rigidity of a rod subjected to extension:

$$\frac{N}{\left(\frac{M}{k}\right)_{k=0}} = \frac{\left(\frac{1}{K^2} - \frac{1}{D^2}\right)A_0}{I_0 - (I_0 - S_0)\frac{K^2}{D^2}},\tag{42.35}$$

where N is the resultant normal force on the ends, M is the torque, S_0 is the torsional rigidity according to the linear theory, I_0 is the moment of inertia of the unstrained cross-section about the Z -axis, and A_0 is the area of the unstrained cross-section. For the incompressible body, put $K^2 D = 1$. (42.32) is included as a special case. These results have been shown to remain unchanged when the cylinder is subjected also to hydrostatic pressure.¹⁸

P. 193, footnote 17, replace "§§3-4" by "§3".

P. 193, after footnote 17, add:

¹⁸ A misleading conclusion obtained in [1951, 15, §4] is corrected in [1952, 16, Part III, Appendix].

P. 193, footnote 3, add: The form assumed by the third order terms in a power series for Σ is determined by BIRCH [1947, **31**, §4] for cubic crystals; by MURNAGHAN [1951, **22**, Ch. 5] for all types of symmetries.

P. 194, l. 1 above (43.4), replace "Mooney's" by "the Mooney-Rivlin".

P. 194, footnote 6, replace first sentence by:

* The result is given by MOONEY [1940, **7**, eq. (47)] in the form $\Sigma = \Sigma_{11} K^1 + \Sigma_{2J} L^J$, where $K^1 \equiv \Sigma_{r=1}^3 [(1 + \delta_r)^{21} + (1 + \delta_r)^{-21} - 2]$, $L^J \equiv \Sigma_{r=1}^3 [(1 + \delta_r)^{2J} - (1 + \delta_r)^{-2J}]$.

P. 195, footnote 7, after "L. BRILLOUIN" insert [1925, **5**, §10], and at the end replace the period after "series" by a semi-colon and add: later [1947, **31**, §5] he generalizes (44.2) to the case of a cubic crystal.

P. 202, l. 6 of first footnote, before "GARCÍA" insert: TEOFILATO [1939, **24**, §5] and.

P. 202, last l. of first footnote, add: The purely elastic solution for the hollow sphere subject to internal pressure given in [1948, **58**] seems to belong also to this theory.

P. 203, footnote 7, l. 4 from end, replace [1948, **42**] by [1947, **32**].

P. 205, footnote 13, before "[1941, **10**]" insert: "[1939, **26**]: a stretched weightless string loaded by a single mass is set into oscillation;" and at the end insert: "[1947, **32**]: longitudinal waves."

P. 206, at the end of §49 add:

The quantitative Kelvin and Poynting effects obtained in §42 consist in additional loads required in order to produce a given deformation—loads acting on surfaces which would be unstressed according to the linear theory. It follows qualitatively that if these new loads be wanting, a new type of deformation, not present in the linear theory, will result. Since the Kelvin and Poynting effects are governed by coefficients different from those which govern the classical effects (§10), we should expect that the magnitudes of these new deformations can be approximated by those which would result according to the linear theory, should the wanting loads be supplied. That such is indeed the case in simple shear and torsion of a circular cylinder is shown by Murnaghan,²⁰ and Rivlin²¹ has proved that the method is entirely general. In the second order theory in which all powers of the displacement gradients higher than the second are systematically neglected, a solution (of course not unique) to an arbitrary stress boundary value problem may be obtained by these steps:

1. Solve the same problem in the linear theory.
2. Calculate the additional loads required according to the second order theory in order to maintain the displacement obtained from Step 1.
3. In the linear theory, find the displacement which results from the loads obtained in Step 2.
4. Add the negatives of the displacements from Step 3 to the displacement from Step 1.

Thus the solution of a second order problem is reduced to the solution of two linear problems and one inverse problem. Doubtless the method could be extended to the analogous n^{th} order approximation and made the basis of a proof of existence of solution.

P. 206, first footnote, add at end: Using one superfluous coefficient, SAKADI [1940, **15**] obtains within this theory solutions for longitudinal and transverse waves, compression and

stretching of a straight bar, compression of a spherical or cylindrical shell, and torsion of a circular cylinder; in [1949, 60], bending of an elliptic cylinder, torsion of an elliptic cylinder, radial vibration of a sphere, torsional vibration of a circular cylinder.

P. 206, footnote 19, at the beginning insert [1925, 5, §§5-12], and at the end add: Previously BORN & MISRA [1940, 17] had calculated the coefficients of the third order terms for certain types of crystals according to a hypothetical law of intermolecular force. As has been remarked by BIRCH [1947, 1, §3], the application of these results by FÜRTH [1942, 20] is vitiated by his use of (40.4) instead of stress-strain relations valid for finite strain.

P. 206, after footnote 19, insert:

²⁰ [1951, 22, Ch. 6, §2, and Ch. 7, §3].

²¹ [1953, 4]. RIVLIN's examples are: the extension and torsion of a cylindrical tube, the extension and torsion of a prism.

P. 206, footnote 1, instead of "angles" read "changes of angle"; before "RICHTER" insert: TROFILATO [1939, 24, §§3-4] shows that formulae expressing the stress components as linear functions of the extensions and changes of angle cannot be tensorially invariant.

P. 206, footnote 2, add at end: Simple extension in an incompressible elastic material for which one of several linear relations holds precisely is discussed in [1951, 21].

P. 207, footnote 6, l. 2 from end, after [1946, 19, §§1-3] insert [1948, 59].

P. 208, replace footnote 9, by:

⁹ [1939, 14]. Solutions, often very elaborate, which concern various types of torsion, bending, and extension of bars of various cross-sections are obtained within this theory in [1938, 1, 12] [1939, 3, 14, 16, 23, 28] [1941, 19-20] [1942, 19] [1943, 16, 20, 22-23] [1944, 18-19] [1947, 24].

P. 208, footnote 2, replace period at end by semi-colon and add: it was proposed in the same form and in the same connection by MAXWELL in his letter of 18 Dec. 1856 to Lord KELVIN [1937, 10, pp. 32-3].

P. 209, footnote 10, add: This theory is proposed anew in [1952, 18].

P. 209, footnote 12, before the first sentence insert: That the expression in question is not a scalar is evident, since it changes sign when 1 and 3 are permuted.

P. 209, footnote 12, l. 2, replace "by" by comma and add: as may be verified by choosing $f(p, q, r) = 3r/(p - q)$, $l = 2$, $m = -1$, $n = -1$ in.

P. 209, footnote 12, l. 3, insert comma after "(9.4)".

P. 210, after (C) of footnote 1, add:

The same results are derived by Novozhilov [1948, 60, §32].

P. 210, footnote 14, after [1947, 8] add [1949, 61].

P. 210, at end of footnote 15, add: Cf. §72^{10, 14}.

P. 211, replace the last sentence of §52 by:

For the case of hydrostatic pressure (52.2) becomes

$$p = \frac{3\lambda_E + 2\mu_E}{2} \left(\frac{v_0}{v} \right)^{\frac{2}{3}} \left[\left(\frac{v_0}{v} \right)^{\frac{2}{3}} - 1 \right]. \quad (52.3)$$

According to Birch⁶ this formula fits rather well many of Bridgman's measurements on the compressibilities of solids at very high pressures (but cf. §57).

P. 211, footnote 5, delete last line and add:

⁶ [1938, 5] [1947, 31, §7] [1952, 17, pp. 242-245]. BIRCH considers also the effect of retaining the terms of third order in the power series for the strain energy for cubic crystals. He concludes that in most cases (52.3) agrees with the data at least as well as the result so obtained. He calculates also the speeds of propagation of infinitesimal dilatational and distorsional waves in a body subjected to large hydrostatic pressure [1939, 27].

P. 214, l. 6 of §54, replace “elasticity” by “elasticity.^{1a}”.

P. 214, before footnote 1, insert:

^{1a} Early experimental results on the elasticity of rubber, expressed in terms of the concepts of the infinitesimal theory, are given in [1871, 6] [1876, 4] [1889, 6]. A simple theoretical application was made by BAKER [1900, 6], who by letting Young’s modulus be a linear function of the square of the length of a cord showed that large extension will effect very little increase in its fundamental frequency of vibration (cf. also [1939, 26].)

P. 215, footnote 6, l. 3 should read:

7, §4]. WALL [1942, 5, pp. 482–488] [1943, 2, p. 530] and TRELOAR [1943, 3, pp. 37–38] give a sepa-

P. 215, footnote 8, in first list of references add: [1946, 11, Ch. 8, §8], and at end add: ISIHARA, HASHITSUME, & TATIBANA [1951, 19] have succeeded in deriving from molecular hypotheses a result equivalent to retaining three terms in (43.4), with special relations connecting the coefficients. WANG & GUTH [1952, 14] obtain a result which is somewhat more general, but still not adequate to fit the results of experiments at moderate strains.

P. 216, l. 3 from end of text, replace period by semi-colon and add: this special case was shown to be inadequate by the experiments of Gee.^{15a}

P. 216, footnote 15, delete last sentence, and add:

^{15a} [1946, 22]. Further experiments on the extension of rubber are discussed in [1950, 16] [1951, 2, §§9–10] [1952, 6].

P. 217, the heading of section IVD should read: *Rate of Deformation Theories*

P. 218, last line, replace “infinitesimal,” by “infinitesimal,^{7a}”.

P. 218, footnote 7, line 3 from end, after [1942, 10] add [1950, 26].

P. 218, after footnote 7, add:

^{7a} GREEN, RIVLIN, & SHIELD [1952, 8] make the stronger assumption $x^i = y^i + \epsilon u^i$, where powers of ϵ higher than the first are to be neglected systematically. Thus they suppose that not only its gradients, but also the second displacement itself is infinitesimal with respect to the first. Restricting attention to isotropic bodies, they obtain stress strain relations considerably simpler than (55.4). They work out also the specializations appropriate to the case when the first deformation is homogeneous, remarking that some of the results of §42J might be obtained in this way. They apply their formulation to problems concerning a semi-infinite plane indented by a rigid punch; to the small bending of stretched plates; to small generalized plane stress superposed on finite stretching.

P. 220, line 3 before end of §55, through p. 224, delete and replace by:
route of attack has been therefore the abandon the idea of a natural state, either partially⁹ or wholly. Murnaghan¹⁰ notices that while indeed a natural state is usually employed in motivation of the infinitesimal theory, the Hooke-Cauchy law (1.1) is also a first approximation to a quite different concept. Starting with a body subject at time t_0 to any state of stress ^{1t}, he regards the present configuration as “built up from a succession of infinitesimal strains by a method of integration,” the stress increments being computed from the strain increments in the limit by a relation having the form of the ordinary Hooke-Cauchy law (1.1) when the body is unstressed. While he does not give any general formulation, limiting his attention to special cases, general theories of this type have been formulated by Jaumann¹¹ and by Hencky¹². These latter do

not appear to be dynamically admissible, however. Before developing a proper substitute, we pause to consider an invariance requirement which any such theory must satisfy.

⁹ A purely formal definition of a "source" of initial stress is given in [1931, 3]; while stress-strain relations are abandoned, the equations proposed are formal analogues of those of linear elasticity theory.

¹⁰ [1944, 6, 13] [1945, 11] [1949, 6, §3]. I am indebted to Professor MURNAGHAN for use of the last paper in MS. It is rather difficult to determine precisely what is assumed and what is not in MURNAGHAN's latest papers. Thus in [1949, 41] he refers to his work on large hydrostatic pressure rather perplexingly as "some results of a comparison of the exact formula [i.e. BOUSSINESQ's formula (39.2)₂] with experiment".

¹¹ [1911, 5, §IX]. In our notation JAUMANN's equation (38) is $\dot{\Psi}^i_j + r^{i,j}_i \bar{d}^l_k = 0$, where $(\alpha + 3\beta) \bar{d}^l_k \equiv \alpha d^l_k + \beta d^m_n \delta^l_k$; his equation (43) is $t^i_j = \Psi^l_{k\eta^k i^m n} r^m_i j - \frac{1}{2} \Psi^l_{k\eta^k i^m n} \Psi^m_n \delta^l_j$, where $\alpha, \beta, r^{i,j}_i, \eta^k i^m_n$ are material constants, and Ψ^i_j is to be eliminated between the two equations. By a method of power series expansion, JAUMANN shows that his theory yields (1.1) as a linear approximation, and concludes that the consequences of his theory and of the classical infinitesimal theory are the same within the range of experimentally observable phenomena. Cf. [1917, 5, §§21-22] [1918, 1, §§96-100].

¹² The method of HENCKY [1921, 2, §2] is as follows. Let a relation for a measure m of strain in terms of the stress t be given. To eliminate the influence of the natural state, differentiate materially, obtaining a power series for $\dot{m}$ in terms of t and $\dot{t}$. Eliminate $\dot{m}$ by a kinematical formula giving $\dot{m}$ in terms of m, d , and possibly w ; then eliminate m wherever it occurs by means of the stress-strain relations. We now have a power series in $t, \dot{t}$, and possibly also $x^i_{,L}$. If all but the linear terms in this power series be neglected, (56.1) results, with $\bar{t}$ replaced by $\dot{t}$.

55 bis. Cauchy's flux principle. Now in any dynamical model of actual matter a rigid rotation should be insufficient to change the state of stress relative to co-ordinates fixed in the body. Equivalently, the absolute flux $(t_{(n)} da)'$ of the resultant force of the stress upon any surface element da must vanish in a motion such that $d = 0$. Now by (20.6), (26.1)₂, and (22.16) we get

$$\begin{aligned} (t^i_{(n)} da)' &= \overline{t^i_{(n)}} da - \dot{x}^i_{,k} t^k_{(n)} da, \\ &= \overline{t^{ij}} da_j - \dot{x}^i_{,k} t^{kj} da_j, \\ &= (\dot{t}^{ij} - t^{ik} \dot{x}^j_{,k} - t^{kj} \dot{x}^i_{,k} + t^{ij} \dot{x}^k_{,k}) da_j. \end{aligned} \quad (55.b1)$$

Put $\bar{t}$ for the *relative time flux* of t :

$$\bar{t}^{ij} \equiv \dot{t}^{ij} - t^{ik} \dot{x}^j_{,k} - t^{kj} \dot{x}^i_{,k} + t^{ij} \dot{x}^k_{,k}. \quad (55.b2)$$

Then we have shown that in order for $(t_{(n)} da)'$ to vanish for arbitrary da , it is necessary and sufficient that $\bar{t} = 0$. Since ideas and analysis essentially equivalent to the foregoing were used by Cauchy¹ in his continuum derivation of (55.4), we shall express our result as *Cauchy's flux principle*: if the defining equations for an ideal material involve the stress rate $\dot{t}$, they shall do so only through the relative

time flux $\dot{\mathbf{t}}$. For if the time rate of change of stress is to affect the dynamics of the substance, it must do so only in a way which is invariant under rigid motions.

Another way of reaching Cauchy's flux principle is to differentiate (26.9)₃ materially, first supposing the reference configuration fixed, thus obtaining

$$\dot{t}^{ij} = J^{-1} x^i_{,\alpha} x^j_{,\beta} \dot{T}^{\alpha\beta}. \quad (55.b3)$$

Consequently $\dot{\mathbf{t}}$ bears the same relation to $\dot{\mathbf{T}}$ as $\mathbf{t}$ to $\mathbf{T}$ (cf. §26¹¹).

¹ [1829, 1]. Similar ideas were employed by ZAREMBA [1903, 9-10] [1937, 6, Ch. I, §2]. I am indebted to Dr. ERICKSEN for pointing out the errors in my original version of §§56-58 and for discussion of this material.

56. Defining relations for rate of deformation theories. The simplest incremental law of pure elasticity satisfying Cauchy's flux principle is

$$\dot{t}^i_j = A^{i\ k}_{j\ l} d^l_k, \quad A^{ijk\ l} = A^{jik\ l}, \quad (56.1)$$

yielding a *rate of deformation theory*¹ in which any stress components t^i_j satisfying Cauchy's equations (26.2) are admissible as initial values for the solutions of (56.1). The coefficients $A^{i\ k}_{j\ l}$ may be supposed to be functions of $\mathbf{t}$. For an infinitesimal displacement we may put $\mathbf{x} \approx \mathbf{u}\ dt$, $\mathbf{e} \approx \mathbf{d}\ dt$, $\mathbf{t} \approx \mathbf{t} + \dot{\mathbf{t}}\ dt$, and there results by (55.b2) Cauchy's formula (55.4)₁ for the stress increment in an initially stressed medium—with the important difference that the coefficients $C^{i\ k}_{j\ l}$ are no longer given by (55.14)₂, but are replaced by the arbitrary material constants $A^{i\ k}_{j\ l}$. Thus, apart from the fact that isotropy has a different meaning, the rate theory also yields the classical theory of initial stress as a first approximation. Isotropy, however can now be defined with respect to the present configuration, all notion of a natural state having been abandoned; i.e., a body is now *elastically isotropic* if $\dot{\mathbf{t}}$ be an isotropic function of $\mathbf{d}$:

$$\dot{t}^i_j = \lambda_E d^k_k \delta^i_j + 2\mu_E d^i_j. \quad (56.2)$$

If the displacement gradients also be infinitesimal, we may approximate (56.2) by²

$$\dot{t}^i_j = \lambda_E d^k_k \delta^i_j + 2\mu_E d^i_j. \quad (56.3)$$

It is now the Hooke-Cauchy law (1.1) which results as a first approximation when the displacement is infinitesimal.

Suppose more generally that $\dot{\mathbf{t}}$ be an isotropic but not necessarily linear function of $\mathbf{d}$:

$$\dot{t}^i_j = \mathfrak{G}_0^{(\mathbf{d})} \delta^i_j + \mathfrak{G}_1^{(\mathbf{d})} d^i_j + \mathfrak{G}_2^{(\mathbf{d})} d^i_k d^k_j. \quad (56.4)$$

As in the classical theory, suppose that there are *only two dimensional moduli* (§47), a *natural elasticity* $\mu_{E\mathfrak{n}}$ of dimension $\mathbf{M}\ \mathbf{L}^{-1}\ \mathbf{T}^{-2}$ and a *reference temperature* θ_0 of dimension Θ . For ease of writing henceforth we omit the superscript $\dot{\mathbf{t}}(\mathbf{d})$. Let the coefficients $\mathfrak{G}_\tau$ be functions³ of $I_{\mathbf{d}}$, $II_{\mathbf{d}}$, $III_{\mathbf{d}}$, $I_{\mathbf{t}}$, $II_{\mathbf{t}}$, $III_{\mathbf{t}}$, $\mu_{E\mathfrak{n}}$, θ , θ_0 . Since

$\dim \mathcal{G}_0 = \mathbf{ML}^{-1}\mathbf{T}^{-3}$, it follows that $\mathcal{G}_0/I_d\mu_{En}$ is a dimensionless function of 9 quantities composed of three fundamental dimensions $\mathbf{ML}^{-1}\mathbf{T}^{-2}$, $\mathbf{T}$, and Θ , and hence must reduce to a function of $9 - 3 = 6$ dimensionless ratios of these quantities:

$$\mathcal{G}_0 = \mu_{En} I_d g_0 \left(\frac{I_t}{\mu_{En}}, \frac{II_t}{\mu_{En}^2}, \frac{III_t}{\mu_{En}^3}, \frac{II_d}{I_d^2}, \frac{III_d}{I_d^3}, \frac{\theta}{\theta_0} \right), \quad (56.5)$$

say. Similar reasoning applied to $\mathcal{G}_1$ and $\mathcal{G}_2$ yields $\mathcal{G}_1 = \mu_{En} g_1$, $I_d \mathcal{G}_2 = \mu_{En} g_2$, where the g_r are dimensionless. In order to prevent the stress rate from becoming singular when $\mathbf{d} = 0$ or $\mathbf{t} = 0$ it is now necessary to put $\mathcal{G}_2 = 0$ and to suppose $\mathcal{G}_0$ and $\mathcal{G}_1$ to be independent of II_d and III_d . Thus we obtain a quasi-linear theory characterized by

$$\dot{\mathbf{t}}^i_j = \mu_{En} I_d g_0 \delta^i_j + \mu_{En} g_1 d^i_j, \quad (56.6)$$

where g_0 and g_1 are dimensionless functions of I_t/μ_{En} , II_t/μ_{En}^2 , III_t/μ_{En}^3 , θ/θ_0 . If $\sqrt{II_t}/\mu_{En} \ll 1$, (56.6) may be approximated by

$$\dot{\mathbf{t}}_j = (\lambda_E + \beta_0 I_t) I_d \delta^i_j + 2\mu_E \left(1 + \beta_1 \frac{I_t}{\mu_{En}} \right) d^i_j, \quad (56.7)$$

where λ_E/μ_{En} , μ_E/μ_{En} , β_0 , β_1 are dimensionless functions of θ/θ_0 . The theory resulting from (56.7) will be called the "second approximation theory;" that from (56.2), the "first approximation."

In the examples which follow, the thermal dependence of λ_E and μ_E will be neglected, and we shall take $2\mu_E = \mu_{En}$.

¹ MURNAGHAN employs the misleading terms "local action" and "action at a distance" for the present and the natural state theories, respectively.

² HANDELMAN, LIN, & PRAGER [1947, 9, §§2-4] propose relations similar to an inverse of (56.3) for infinitesimal strains produced by loading in the strain-hardening range, after concluding that finite relations of the type (45.1) for loading in general cannot be combined with (1.1) for unloading. Equations similar to (56.3) but tensorially not admissible for isotropic media are proposed in [1948, 32]. If I correctly understand the work of SWAINGER [1947, 21, §4.1] [1949, 37, §4.1], he puts each component of a certain undefined strain rate (§17¹) separately proportional to the corresponding component of the stress rate $\dot{\mathbf{t}}$; relations of this type are inadmissible for isotropic media unless the factors of proportionality for the separate components are the same. Cf. §60². A pair of elaborate stress rate equations containing his "projective" rates (§21²) are proposed by HENCKY [1949, 40] as "universal equations of rheology".

³ This assumption is not the most general possible, since $\Phi(\mathbf{t}, \mathbf{d})$, $\Phi(\mathbf{t}^2, \mathbf{d})$, $\Phi(\mathbf{t}, \mathbf{d}^2)$, and $\Phi(\mathbf{t}^2, \mathbf{d}^2)$ might well be added to the list.

⁴ The present theory is one of pure elasticity, since there is no "natural time" or "time of relaxation" (§62). In fact, (56.2) results from letting the relaxation time t_n approach ∞ in the equations $\dot{\mathbf{t}}^i_j = \lambda_E I_d \delta^i_j + 2\mu_E d^i_j + t_n^{-1} \{ t^i_j - p \delta^i_j - \alpha(p - \bar{p}) \delta^i_j \}$, which were proposed by ZAREMBA [1903, 9, eq. (32)] as a generalization of MAXWELL's theory of relaxation (§81).

57. Murnaghan's formula for hydrostatic pressure. By putting $t^i_j = -p\delta^i_j$, $3\dot{x}^i_{,j} = 3d^i_j = d^k_k \delta^i_j = \overline{\log v} \delta^i_j$ into (56.6) we get a relation of the form

$$-v \frac{dp}{dv} = f(p), \quad (57.1)$$

obtained by Murnaghan in a different way.¹ In fact

$$f(p) = \frac{1}{3}p + 2\mu_E(g_0 + \frac{1}{3}g_1), \quad (57.2)$$

where the arguments of the g_T are $-3p/2\mu_E$, $3p^2/4\mu_E^2$, $-p^3/8\mu_E^3$. The second approximation theory yields $f(p) = a(p + A)$, where $a = \frac{1}{3} - 3\beta_0 - \beta_1$, $aA = \lambda_E + \frac{2}{3}\mu_E$. In this case by integrating (57.1) we obtain

$$p = A \left\{ \left(\frac{v_0}{v} \right)^a - 1 \right\}, \quad (57.3)$$

where v_0 is the value of v when $p = 0$. *This formula distinguishes between hydrostatic pressure and hydrostatic tension:* for no finite value of p can v become zero but if $p = -A$ then v becomes infinite, so that A is an upper bound for the rupture or yield stress. By choice of A and a , Murnaghan² fits within 1% the results of Bridgman's experiments on the compression of sodium up to pressure of 10^5 atm. If $a = 2$, (57.3) becomes equivalent to Laplace's law for the density of the earth. In the first approximation theory, $a = \frac{1}{3}$.

Comparing (57.1) with its counterpart (42.5) reveals a characteristic difference between the present theory and the classical natural state theory: when f is specified, (57.1) determines the equation of state $v = v(p)$ only to within an arbitrary constant, while the functional form of Σ completely determines the hydrostatic equation of state. Moreover, the striking special case (57.3) arises naturally by approximating (57.1), while, although it is quite compatible with the classical finite strain theory, it does not appear to result from (42.5) by any otherwise natural choice of the form of Σ .

¹ [1944, 6, p. 246] [1945, 11] [1949, 6, §3] [1951, 22, Ch. 4, §3]. Cf. [1952, 15]. GLEYZAL [1949, 38, §3] shows that from the assumption $p\delta^i_j; x^i_{,\beta} X^{\alpha}_{,i} = A\tilde{E}^{\alpha}_{\beta} + B\tilde{E}^{\gamma}_{\gamma}\delta^{\alpha}_{\beta}$, where for $\tilde{E}$ various rather arbitrarily selected combinations of C , $\dot{C}$, and their time integrals are selected, result various laws similar to (57.3). Simple extension is treated in the same tentative spirit.

² [1949, 6, introd.]. Cf. [1944, 6], where by putting $a = 2$ and choosing but the single parameter A a less good fit for the compression of lithium in the same range is obtained. There remains, however, an element of curve-fitting, since MURNAGHAN makes no use of the relation $aA = \lambda_E + \frac{2}{3}\mu_E$, whereby a and A are related to the ordinary LAMÉ coefficients measurable in infinitesimal strains.

58. Simple extension. To treat simple extension Murnaghan¹ appears to abandon the notions of §56 and instead to fall back upon a special case of (55.4), together with further assumptions which I do not understand. I shall derive from (56.6) a result equivalent to his by considering the special motion $x = Xe^{-k\sigma t}$, $y = Ye^{-k\sigma t}$, $z = Ze^{kt}$. Then $\dot{x} = -k\sigma x$, $\dot{y} = -k\sigma y$, $\dot{z} = kz$, $d^x_x =$

$d_y^y = -k\sigma$, $d_z^z = k = \dot{z}/z = \overline{\log z}$, $I_d = (1 - 2\sigma)k = (1 - 2\sigma)\overline{\log z}$. If the inertial forces may be neglected, this motion becomes dynamically possible for a stress $t_x^x = t_y^y = 0$, $t_z^z = I_t$. The former condition put into (56.6) yields $(1 - 2\sigma)g_0 = \sigma g_1$. This relation gives the ratio σ of the lateral contraction rate to the longitudinal extension rate:

$$\sigma = \frac{1}{2 + \frac{g_1}{g_0}} : \quad (58.1)$$

the result depends not only on the elastic constants of the material but also upon the magnitude of the stress t_z^z . In the second approximation theory we get

$$\sigma = \frac{1}{2} \frac{1}{1 + \frac{\mu_E + \frac{1}{2}\beta_1 t_z^z}{\lambda_E + \beta_0 t_z^z}}, \quad (58.2)$$

while in the first approximation, or for sufficiently small stress t_z^z , we recover the classical result $\sigma = \nu_E$. For t_z^z itself we obtain from (56.6)

$$\dot{t}_z^z = \dot{t}_z^z - (1 + 2\sigma)t_z^z \overline{\log z} = 2\mu_E[(1 - 2\sigma)g_0 + g_1]\overline{\log z}, \quad (58.3)$$

whence by (58.1) follows

$$\log(1 + \delta_1) = \log \frac{z}{Z} = \int_{1/t_z^z/2\mu_E}^{t_z^z/2\mu_E} \frac{(2g_0 + g_1)d\xi}{(4g_0 + g_1)\xi + 3g_0 + g_1}, \quad (58.4)$$

where the arguments of g_0 and g_1 are ξ , 0, 0, and where ${}^1t_z^z$ is the initial uniaxial stress. In the second approximation theory the quadrature may be evaluated easily, yielding

$$(1 + \delta_1)^{(E/\mu_E)(1/P+1/Q)} = \left(\frac{1 + \frac{t_z^z}{P\mu_E}}{1 + \frac{{}^1t_z^z}{P\mu_E}} \right)^{1-\frac{1}{2}AP} \left(\frac{1 - \frac{{}^1t_z^z}{Q\mu_E}}{1 - \frac{t_z^z}{Q\mu_E}} \right)^{1+\frac{1}{2}AQ} \quad (58.5)$$

where E is Young's modulus and where the other moduli are derived from the Lamé constants and β_0 , β_1 , as follows:

$$\begin{aligned} A &\equiv \frac{2\beta_0 + \beta_1}{\frac{\lambda_E}{\mu_E} + 1}, & 2B &\equiv \frac{\frac{2\lambda_E}{\mu_E} + 1 + 3\beta_0 + \beta_1}{\frac{3\lambda_E}{2\mu_E} + 1}, & C &\equiv \frac{4\beta_0 + \beta_1}{\frac{3\lambda_E}{2\mu_E} + 1}, \\ \frac{1}{2}P &\equiv \frac{B}{C} \left(1 + \sqrt{1 - \frac{C}{B^2}} \right), & \frac{1}{2}Q &\equiv -\frac{B}{C} \left(1 - \sqrt{1 - \frac{C}{B^2}} \right). \end{aligned} \quad (58.6)$$

Murnaghan's result is equivalent to the case $A = 0$, $t_z^s = 0$. *This result distinguishes between pressure and tension*: Q_{μ_E} is the value of the tension t_z^s which yields infinite stretch, while P_{μ_E} is the value of the pressure $-t_z^s$ which reduces the length to zero. There is thus a qualitative indication of yield or rupture both for extension and for compression. In the first approximation, only one of the two limit phenomena, that for compression, occurs:

$$(1 + \delta_1)^{E/R\mu_E} = \frac{1 + \frac{t_z^s}{R\mu_E}}{1 + \frac{1t_z^s}{R\mu_E}}, \quad R \equiv \frac{3\lambda_E + 2\mu_E}{2\lambda_E + \mu_E}. \quad (58.7)$$

For small extension of an initially unstressed bar, this result reduces to the classical formula (42.8)₁.

A great deal of work remains to be done before this new rate of deformation theory of elasticity can be properly understood and evaluated.

¹ [1944, 13] [1945, 11] [1949, 6, §4].

P. 225, after end of text insert:

Bateman⁶ remarked that the "experimental tests" which supposedly confirm the linear relation (1.2) concern degenerate situations such as rectilinear shearing flow and serve merely to verify the interconnections of certain dimensionless parameters, thus failing entirely to discriminate between the classical linear viscosity and the myriad other possible laws which would give rise to the same relations between the same dimensionless parameters in the same degenerate situations.⁷

P. 226, after end of first footnote add:

⁶ [1932, 2, Part II, §1.7].

⁷ ANDERSSON [1952, 10] shows by simple examples that linear response in Couette or Poiseuille flow does not necessarily imply (1.2) in general.

P. 229, lines 10–11, replace "a spherical . . . loss." by: in a spherically symmetric motion $\dot{\mathbf{x}} = c(t)\mathbf{r}$ the stress system, which in general reduces to a hydrostatic pressure $[-p + (3\lambda_E + 2\mu_E) c] \delta^i_j$, is unaffected by the viscosity of the fluid or the motion taking place, and further, in case $p = p(\rho)$, the motion itself is unaffected by the viscosity.

P. 230, line 21, after [1948, 4, §IVA] add: [1951, 28, §II].

P. 230, line 9 from bottom, between "for" and "major" insert "the".

P. 230, line 4 from bottom, replace [1953, 1] by [1954, 2].

P. 231, line 6 from end of text, replace "time." by "time.^{2a}".

P. 231, after footnote 2, add:

^{2a} This fact is remarked by WHITE (quoted on p. 93 of [1949, 59]).

P. 239, line 1, between "order" and "being" insert: in d.

P. 239, footnote 2, lines 1–2 from end, replace "REINER" by "BRAUN & REINER", "him" by "them", "his" by "their".

P. 241, line 6 from end of text, replace "Reiner⁶ remarks" by "Braun & Reiner⁶ remark".

P. 241, footnote 7, add: [1950, 25].

P. 242, footnote, add at end: The effect of variable viscosity and thermal conductivity is considered by LIN & STREET [1953, 6].

P. 243, line 7 from end of text, replace "Nisson" by "Nissan".

P. 243, footnote 9, add: [1947, 33, p. 79] [1950, 25]. The last paper gives a history of the experimental discovery of the phenomena discussed in this section [p. 55]. Actually, once the effect was noticed, it was recognized as familiar: e.g., rotary stirrers are useless for paint and have to be replaced by other means of agitation. What is new is the simple mechanical explanation.

P. 243, footnote 10, delete line 4 through end and replace by:
and BRAUN & REINER [1952, 5] propose experiments to decide whether the effect arises from elastic or viscous properties. To me the issue seems not to exist: the theory of elasticity is applicable only to substances in which the stress does not subside until recovery of strain from some specified configuration; while the physical fluids used in the experiments are visco-elastic, the phenomena in question pertain to steady motions (cf. [1948, 5, §17]), and an attempt to represent such phenomena by a theory of elasticity seems unnecessary, irrelevant, and inappropriate. There is however, a sort of reconciliation of the two views in the fact that the REINER-RIVLIN fluid possesses a natural elasticity, say $\mu\nu^2/\mathcal{G}_{2000}$ (§§67, 70). The possible effects of elastic stresses in these phenomena have been the subject of a controversy between OLDROYD [1950, 3, Added note] and GREENSMITH & RIVLIN [1953, 4, §2].

P. 243, footnotes 11 and 12, replace "REINER" by "BRAUN & REINER".

P. 244, line 1 after (72.16), replace "conditions . . . are" by: (72.16)₂ yields

$$\omega' \mathcal{G}_1(0, -\frac{1}{4}r^2\omega'^2, 0) = f(r), \quad (72.17a)$$

while the condition of integrability for (72.16)₁ and (72.16)₂ is

P. 244, delete (72.17)₂ and (72.17)₃, renumber (72.17)₁ as (72.17b); delete (72.18)₁, and in (72.18) replace the exponent " $-\frac{1}{2}$ " by " -3 ".

P. 244, line 1 after (72.18), delete " $C = 0$ ".

P. 245, line 7, replace "Reiner¹³ states" by "Braun & Reiner¹³ state".

P. 245, after line 9 add: Greensmith & Rivlin¹⁴ present very careful experiments, together with corrections for centrifugal forces and edge effects, from which it appears that there is full quantitative accord between experiment and the general solution (72.19)₂, though the second approximation (72.21) is insufficient.

P. 245, footnote 13, add: Cf. also [1950, 25].

P. 245, after footnote 13, add:

¹⁴ [1951, 24] [1953, 4]. However ROBERTS [1952, 20] reports that his own somewhat similar experiments confirm WEISSENBERG's theory (§51) and contradict the REINER-RIVLIN theory.

P. 246, footnote 9, after [1932, 6, Ch. VIII] add: [1933, 11, §§2-3].

P. 247, footnote 12, add: An exhaustive study of the solutions representing plane shock waves which may be obtained from the Chapman-Enskog method is given by ZOLLER [1951, 26].

P. 251, footnote 4, add: The pressure gradient term is noticed also by LEONTOVICH [1936, 6, §2], who states that its coefficient is zero for a monatomic gas according to the kinetic theory.

P. 253, at the end of the first paragraph of §80 add: These results of Greenspan cast grave doubt both on the necessity for modifying the Navier-Stokes equations at all, as far as rarefied monatomic gases are concerned, and on the validity of all kinetic theory methods used so far. With singular disregard for the results of experiment, Wang Chang & Uhlenbeck^{6a} persevere in their positive assertion that the experiments of Greenspan "go beyond the range of the Stokes-Navier and Burnett approximations . . ." Further unpublished results of Greenspan give additional confirmation of the correctness of the Navier-Stokes equations for ultrasonic waves at extremely large values of $\mathcal{C}$.

P. 253, footnote 1, add: Earlier ROCARD [1933, 11, §4] had considered the effect of some of the terms involving density gradients.

After footnote 6, add:

^{6a} [1952, 3]. In this paper WANG CHANG & UHLENBECK persist in using the method of series expansion for solution of the frequency equation, although GREENSPAN [1950, 23] has remarked that the series in question diverge for part of the range of his experiments; WANG CHANG & UHLENBECK, indeed, appear to despair [p. 38] of obtaining results which can be compared with experiment. That their claim to show theoretically the failure of the Navier-Stokes equations rests on confusion of sufficiency with necessity is remarked in [1952, 7].

P. 253, footnote 7, replace [1953, 1] by [1954, 2].

P. 254, last line of text, replace "*constant*." by "*constant*.^{7a}"

P. 254, footnote 7, line 2 from end, put a period after [1903, 9, eq. (32)], delete the remainder of the footnote, and add: In [1937, 6, Chs. II, III] ZAREMBA discusses acceleration waves and various types of Couette flow in this theory.

P. 254, after footnote 7, add:

^{7a} REINER analyses the effect of the natural time as a relaxation time in the case of hydrostatic pressure. I am obliged to him for use of his paper [1953, 2] in MS.

P. 255, footnote 8, in the first list of references add: [1935, 5-7] [1939, 25] [1940, 16] [1946, 11, Ch. IV, §§7-8, 10] [1949, 59] [1950, 27] [1951, 18, 25].

P. 260, footnote 1, delete "the".

Corrections to the list of references:

1904, 2. The title should read: *Właściwa praca odkształcenia jako miara wytężenia materiału*, Czas. techn. (Lwow) 22, 38-40, 49-50, 61-62.

1942, 12. Replace "*generle*" by "*generale*".

1944, 19. Replace "*rastyazsheniya*" by "*rastyazheniya*".

1946, 20. Replace "NISSON" by "NISSAN".

1947, 19. Add: Abstract in Ber. Math.-Tagung Tübingen 1946, 109-110.

1948, 42. Delete.

1951, 11. Replace "Z. . . forthcoming" by "(in Hebrew), Hebrew Inst. Tech. Haifa Sci. Publ. 4 (1950), 15-30".

1952, 2. Replace "forthcoming" by "79-95".

1952, 3-4. Delete.

1952, 5. Replace "M. REINER" by "I. BRAUN & M. REINER", "Math. . . forthcoming" by "Mech. Appl. Math. 5, 42-53".

Additions to the list of references:

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Corrections and additions for existing names in the index of authors:

BATEMAN, add 59 ⁶	FRESNEL, add 3
BIRCH, delete 52 ⁵ ; add 43 ³ , 49 ¹⁰ , 52, 52 ⁸ , [1939, 27] [1947, 31] [1952, 17]	GARNER, add 727 ¹³ , [1947, 33] [1950, 25]
BOONSTRA, replace 54 ¹⁵ by 54 ^{15a}	GORGIDZE, add [1941, 19–20] [1943, 22]
BORN, add 49 ¹⁰ , [1940, 17]	GREEN, A. E., add 42 ^{17–18} , 55 ^{7a} , [1952, 8]
BRILLOUIN, L., add [1925, 5]	GREENSPAN, add 80 ^{6a}
CAUCHY, delete 14 ³ ; add 14 ⁴ , 19 ^{2a} , 55b ¹	GUTH, add 54 ⁸ , [1952, 14]
DORN, replace 56 ⁵ by 56 ²	HANDELMAN, replace 56 ⁵ by 56 ²
ERICKSEN, delete 57; add 39 ² , 41 ^{9a} , 55b ¹ , [1953, 5]	HENCKY, delete 56 ⁵ ; add 55, 55 ¹²
FINCK, add [1947, 29] [1949, 56] [1951, 17]	JAUMANN, replace 56, 56 ³ by 55, 55 ¹²
FRENKEL, delete 3; add 54 ⁸ , 81 ⁸ , [1940, 16]	KRYLOV, add [1948, 59]
	LAMB, add 22 ⁸ , [1877, 2]
	LATTER, replace 56 ⁵ by 56 ²

- LEONTOVIČ, add 76⁴, [1936, 6]
 LIN, C. C., replace 56⁸ by 56²
 LOHR, replace 56³ by 55¹¹
 MAXWELL, add 6⁷, 51², [1937, 10]
 MILNE-THOMSON, add [1952, 13]
 MURNAGHAN, delete 14³, 56, 56⁴; add 49²⁰, 55¹⁰, [1951, 22]
 NISSON, correct to NISSAN, add 72⁷, ¹³, [1947, 33] [1950, 25]
 OLDROYD, add 72¹⁰, [1951, 25]
 OSIDA, add [1950, 27]
 PANOV, add [1939, 28]
 PRAGER, replace 56⁵ by 56²
 REINER, add 81^{7a}, ⁸, [1951, 18] [1953, 2]
 RICHTER, add 35¹⁰, [1952, 9]
 RIVLIN, delete 54¹³⁻¹⁷; add 42¹⁸, 49, 49²¹, 54¹³⁻¹⁴, 15a-17, 55^{7a}, 72¹⁰, ¹⁴, [1951, 24] [1952, 6, 8, 16] [1953, 4]
 RIZ, add 42^{9a}, [1938, 12]
 ROCARD, add 80¹, [1933, 11]
 RUKHADZE, add [1941, 19-20] [1943, 23]
 SAKADI, add 49¹⁶, 55⁷, [1940, 15]
 SAUNDERS, replace 54¹⁵ by 54^{16a}; add [1952, 6]
 SCOTT-BLAIR, add 81⁸, [1949, 59]
 SETH, delete [1948, 42]; add 54^{1a}, [1939, 26] [1947, 32]
 SHIELD, R. T., add 42¹⁷⁻¹⁸, 55^{7a}, [1952, 8]
 SWINGER, replace 56⁸ by 56², add [1947, 30]
 TOUPIN, delete 57, add 14⁴
 TRUESDELL, delete [1952, 1-4]; add 57¹, 80^{6a}, [1952, 1-2, 7, 15] [1953, 1] [1954, 1-2]
 UHLENBECK, add 80^{6a}, [1951, 28] [1952, 3]
 WANG CHANG, add 80^{6a}, [1951, 28] [1952, 3]
 WEBER, add 42^{9a}, [1921, 3]
 WEDDERBURN, add 6^{3a}
 WEISSENBERG, replace 14³ by 14⁴, add [1949, 61]
 WHAPLES, add 28¹, [1952, 12]
 ZAREMBA, add 55b¹
 ZVOLINSKI, add [1939, 23]

Additional entries for the index of authors:

- ANDERSSON, B. 59⁷, [1952, 10].
 ARIANO, R. 4¹, 14^{6a}, 41⁹, ¹³, 42^{1a}, 50² [1924, 8] [1925, 6] [1928, 8] [1930, 19] [1933, 12] [1934, 5] [1951, 21].
 ASHWELL, D. G. 42^{9a}, [1951, 23].
 BAKER, T. J. 54^{1a}, [1900, 6].
 BAKHSHIYAN, F. A. 49², [1948, 58].
 BALANKIN, I. A. 42^{9a}, [1927, 6].
 BOYER, C. B. 27¹, [1943, 1].
 BRAUN, I. 72², ⁶, ¹⁰⁻¹³, [1952, 5].
 BUCKLEY, J. C. 42^{9a}, [1914, 3].
 CAFFYN, J. E. 81⁸, [1949, 59].
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 CULLIMORE, M. S. G. 42^{9a}, [1949, 58].
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 GREENSMITH, H. W. 72, 72¹⁰, ¹⁴, [1951, 24] [1953, 4].
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P. 299, the title of IVD should read: Rate of Deformation Theories.

P. 299, the title of §56 should read: Defining relations for rate of deformation theories.